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Three algebras arising from Multiple Zeta Values

by

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Leonhard Euler (1707–1783) investigated the values of the numbers

$$\zeta(s)=\sum_{n\geq 1}\frac{1}{n^s}$$

for s a rational integer, and Bernhard Riemann (1826–1866) extended this function to complex values of s. For s a positive even integer, $\zeta(s)/\pi^s$ is a rational number. Our knowledge on the values of $\zeta(s)$ for s a positive odd integer is extremely limited. Recent progress involves the wider set of numbers

$$\zeta(s_1,\dots,s_k)=\sum_{n_1>n_2>\cdots>n_k\geq 1}\frac{1}{n_1^{s_1}\cdots n_k^{s_k}}$$

for $s_1,\dots,s_k$ positive integers with $s_1\geq 2$. Some Bourbaki lectures (by Pierre Cartier in March 2001 and by Pierre Deligne in January 2012) have been devoted to this question. As a matter of fact, there are three **Q**–algebras which are intertwined: the first one is the subalgebra of the complex numbers spanned by these multizeta values. Another one is the algebra of formal multizetas arising from the known combinatorial relations among the multizeta values. The main conjecture is to prove that these two algebras are isomorphic. The solution is likely to come from the study of the third algebra, which is the algebra of motivic zeta values, arising from the pro–unipotent fundamental group, involving cohomology, mixed Tate motives. Outstanding progress (mainly by Francis Brown) has been made recently on motivic zeta values.

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