Abstract

A series of the form $\sum_{n=1}^{\infty} f(n) \frac{q^n}{1-q^n}$, with $|q| < 1$ for complex q is called a Lambert series. The Lambert series is related to a power series in q in the following manner: $\sum_{n=1}^{\infty} f(n) \frac{q^n}{1-q^n} = \sum_{n=1}^{\infty} q^n \sum_{d|n} f(d)$. Through the use of formal power series, we may relate the coefficients of the product of two Lambert series to the convolution sum of divisor functions $\sum_{m=1}^{n-1} \sigma_a(m)\sigma_b(n-m)$, where $n \in \mathbb{N}$, and $\sigma_a(n) = \sum_{d|n} d^a$. Using an identity given by Alaca, Alaca, McAfee and Williams, we may rewrite the product of two Lambert series as a power series in q. In this manner, we derive a recursive formula for the convolution sum of divisor functions.