Title: A two-dimensional theta function
Speaker: Kenneth S. Williams (Carleton University)
Time: Friday October 1 at 2:30pm
Place: Macphail Room, Carleton University
Abstract: Let k, ℓ, m be positive integers. We define the two-dimensional theta function

$$\Phi_{k,\ell,m}(q) := \sum_{r,s=-\infty}^{\infty} (r\sqrt{\ell} + s\sqrt{-m})^{2k} q^{\ell r^2 + ms^2}, \quad q \in \mathbb{C}, \quad |q| < 1.$$

We are interested in expressing $\Phi_{k,\ell,m}(q)$ as a rational linear combination of products of powers of q and the infinite products E_k $(k \in \mathbb{N})$, which are defined by

$$E_k = E_k(q) := \prod_{n=1}^{\infty}(1 - q^{kn}).$$

It is a classical result of Klein and Fricke (1892) that

$$\Phi_{1,1,4}(q) = 2qE_4^6.$$

Recently Chan, Cooper and Liaw (2009) reproved this result and established that

$$\Phi_{1,1,3}(q) = 2qE_2^3 E_6{}^3.$$

We use theta functions and Eisenstein series to prove some general results, which enable us to determine many identities of this kind, for example we show that

$$\Phi_{1,1,2}(q) = 2qE_1^2 E_2 E_4 E_8^2$$

and

$$\Phi_{1,2,3}(q) = 4q^2 \frac{E_4^3 E_6^7 E_{24}^2}{E_3^2 E_{12}^4} - 6q^3 \frac{E_3^2 E_4^5 E_6^3 E_{24}^4}{E_2^2 E_8^2 E_{12}^4}.$$

This is research in progress.