MAT 1308 A Assignment 1 (Due TUE. Jan. 25th, 17:30) **Student Number:**

NAME:

Problem 1: What is TRUE and what is FALSE? Circle the true statements!

(a) $-\frac{1}{5} = \frac{-1}{5} = -(\frac{1}{5}) = \frac{1}{-5}$

(b) $(\frac{-1}{3})^2 = 3^2$

(c) $\frac{4^2}{4} = 2$

(d) $\frac{1\times 3}{2\times 3} = \frac{1}{2}$

(e) $[-1,\infty) = \{x \in \mathbf{R} \mid -1 < x\}$

(f) $\pi \in \mathbf{Q}$

(g) $1 = \sqrt[3]{-1}$

(h) $\frac{\frac{2}{3}}{\frac{5}{7}} = \frac{2}{3} \times \frac{5}{7}$

(i) $\frac{1}{2} \times \frac{3}{5} = \frac{1\times 5}{2\times 3}$

(j) $\sqrt[2]{(-2)^2} = 2$

(k) $\sqrt[3]{(-2)^3} = 2$

(l) $2^{\frac{1}{2}} = \sqrt{2}$

Problem 2: Fill in the following computations:

(a) $\frac{1}{5} - \frac{3}{2} = \frac{1\times 2}{5\times 2} - \frac{3\times 5}{2\times 5} = \frac{1\times 2 - 3\times 5}{5\times 2} = \frac{\dots}{\dots}$ $-\frac{13}{10}$

(b) $1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{4+1}{4} = \frac{5}{4}$

(c) $2 - \frac{1}{3} = \frac{2\times 3}{3} - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3}$

(d) $4^{-2} = \frac{1}{16} = \frac{1}{4^2}$

Problem 3: Fill in by using one of the symbols: $\bigcap, \bigcup, \subset, \in, \notin, \supset, \not\subset$.

$\cap$ $(-\infty, 0) \dots [1, +\infty) = \emptyset$.

$\in$ $2011 \dots \mathbf{N}$.

$\notin$ $\frac{1}{2} \dots \{1,2,3,4,5\}$.

$\cap$ $\{-1, \frac{-1}{5}, 0, 3, \sqrt{2}\} \dots \mathbf{Q} = \{-1, \frac{-1}{5}, 0, 3\}$.

$\cap$ $(-3, 1) \dots (0, 5) = (0, 1)$.

$\subset$ $\{2\} \dots \mathbf{N}$.

$\in$ $0.15 \dots \mathbf{Q}$.

$\not\subset$ $\{0.15\} \dots \mathbf{N}$.

Problem 4: Compute:

(a) $\frac{1}{1+\frac{1}{1+3}} =$ $\frac{1}{1+\frac{1}{4}} = \frac{1}{\frac{4}{4}+\frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

(b) $\frac{1}{x+2} + \frac{2}{x-1} =$ $\frac{(x-1)\cdot 1 + (x+2)\cdot 2}{(x+2)(x-1)} = \frac{x-1+2x+4}{(x+2)(x-1)} = \frac{3x+3}{(x+2)(x-1)}$

Problem 5: Rationalize:

$\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} =$ $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \cdot \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3} = \frac{2+3+2\sqrt{6}}{-1} =$

$= -(5+2\sqrt{6})$

Problem 6: Simplify:

$i)(2a^{\frac{1}{2}}b^2)^2$ $ii)\sqrt[2]{16y^2}$

(i) $2^2(a^{\frac{1}{2}})^2(b^2)^2$

$4ab^4$

$\sqrt[2]{16}\cdot\sqrt{y^2}$

$4\sqrt{y^2}$

$4|y|$

Problem 7: SOLVE:

(a) $1 + |1 - 2x| > 2$.

(b) $\frac{1}{x+3} \leq \frac{1}{x-2}$.

(c) $\frac{2}{x} = \frac{1}{x} + 2$.

(d) $\frac{(x-2)(3-x)}{2x-1} < 0$.

(e) $x^2 - 3x - 6 > 0$.

(f) $2x - x^2 = 2 + 2x^2$.

a) $|1-2x| > 1$

Case 1°) $\boxed{1-2x \geq 0}$ (So $\frac{1}{2} \geq x$)

Then $1-2x > 1$, So $0 > 2x$;

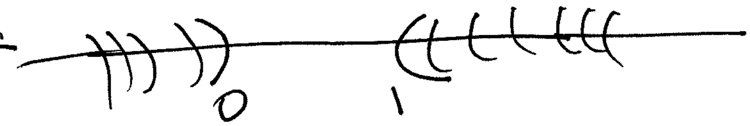
$0 > x$. We get: $\boxed{x < 0}$

Case 2°) $\boxed{1-2x < 0}$ (So: $\frac{1}{2} < x$)

Then: $-1+2x > 1$; $2x > 2$; $x > 1$

We get $\boxed{x > 1}$

conclusion:



b) $0 \leq \frac{1}{x-2} - \frac{1}{x+3}$

$0 \leq \frac{5}{(x-2)(x+3)}$. Since $5 > 0$ we get: $(-\infty, -3) \cup (2, \infty)$

[OUTSIDE the roots ...]

c) $\frac{2}{x} - \frac{1}{x} - 2 = 0 \Rightarrow \frac{2-1-2x}{x} = 0 \rightarrow \frac{1-2x}{x} = 0$

So $1 - 2x = 0 \Rightarrow 1 = 2x \Rightarrow x = \frac{1}{2}$

d)

x	$-\infty$		$\frac{1}{2}$		2		3		$+\infty$
$x-2$		$- - - -$		$- - -$	0	$+$		$+ + +$	
$3-x$		$+ + + +$		$+ + +$		$+$	0	$- - - -$	
$2x-1$		$- - -$	0	$+ + +$		$+$		$+ + + +$	
fraction		$+ + +$	|	$- - -$	0	$+ +$	0	$- - -$	

ANSWER: $(\frac{1}{2}, 2) \cup (3, \infty)$

More space for Problem 7.

(e) Roots: $\frac{3 \pm \sqrt{33}}{2}$; Answer: $\left(-\infty, \frac{3-\sqrt{33}}{2}\right) \cup \left(\frac{3+\sqrt{33}}{2}, +\infty\right)$

(f) $2x - x^2 = 2 + 2x^2$

$\Downarrow$

$0 = 3x^2 - 2x + 2$

$$x_1 = \frac{2 - \sqrt{(-2)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3} = \frac{2 - \sqrt{4 - 24}}{6}$$

Since -20 is NOT positive, we get no roots.

Problem 8: (I) Find the domain for each of the following functions:

(a) $f(x) = x^2 - 1$.

(b) $g(x) = \frac{1}{|x+1|}$

(c) $h(x) = \frac{x}{\sqrt{x^2-1}}$

(d) $k(y) = \frac{|y-1|}{|y|-1}$

(e) $s(x) = \sqrt{x^2 - 2x + 1}$

(f) $p(x) = \frac{x-1}{x-1}$

(II) Find $f \circ g(0)$ et $f \circ k(0)$.

(III) Find the equation of the line that cuts (intersects) the graph of $f(x)$ at $x = 1$ and $x = 2$.

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a) $\mathbb{R}$ b) $|x+1| \neq 0 \Rightarrow x+1 \neq 0 \Rightarrow x \neq -1$

c) $x^2-1 > 0 \Rightarrow (x-1)(x+1) > 0 \Rightarrow (-\infty,-1) \cup (1,\infty)$

d) $|y| - 1 \neq 0 \Rightarrow |y| \neq 1 \Rightarrow \boxed{y \neq 1, y \neq -1}$

e) since $x^2-2x+1 = (x-1)^2 \geq 0 \Rightarrow \boxed{\mathbb{R}}$

f) $x \neq 1$

II $f \circ g(0) = f(g(0)) = f\left(\frac{1}{|0+1|}\right) = f\left(\frac{1}{|1|}\right) =$

$= f(1) = 1^2 - 1 = 0$

$f \circ k(0) = f(k(0)) = f\left(\frac{|0-1|}{|0|-1}\right) = f\left(\frac{|-1|}{0-1}\right) =$

$= f\left(\frac{1}{-1}\right) = f(-1) = (-1)^2 - 1 = 1 - 1 = 0$

III $y = mx + n$; $\underline{m = \text{SLOPE}}$;

$m = \frac{f(2)-f(1)}{2-1} = \frac{2^2-1-(1^2-1)}{1} = 3$; So $y = 3x + n$

But $(1, f(1))$ is on the line. So $f(1) = 3 \cdot 1 + n$

So $n = 0 - 3 = -3$. Then $y = 3x - 3$.

More space for Problem 8.

Problem 9: Let $y = f(x)$ be a function. We shift up the graph of $f(x)$ 2 units, and then we shift (horizontally) 3 units to the left. The resulting graph belongs to the function $g(x) = f(x+a)+b$. FIND a and b.

ANSWERS: $a =$ 3 and $b =$ 2

Problem 10: Find c such that the equation

$$4x^2 - 12x + c = 0$$

has a unique solution.

Condition: $(-12)^2 - 4 \cdot 4 \cdot c = 0$

So: $144 - 16c = 0$

So: $c = \frac{144}{16} = \frac{36}{4} = \boxed{9}$

Problem 11: Below you will find the graph of a function $f(x)$.

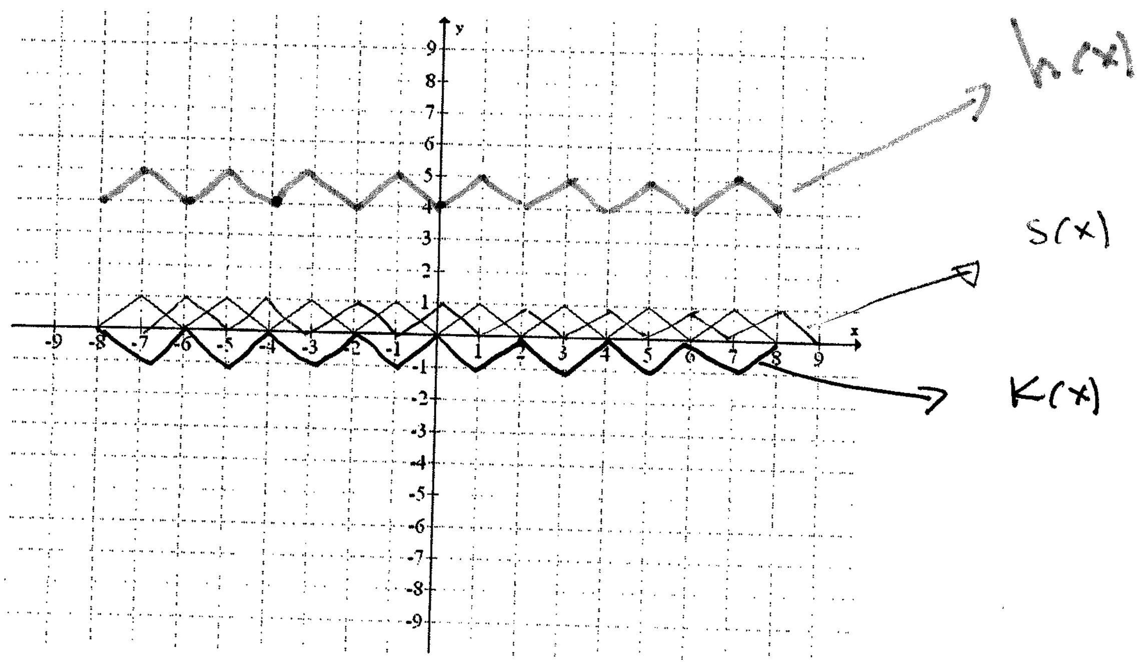
(a) Use this graph to compute: $f(0), f(3)$ and $f(-1)$.

(b) Is f an even function? Justify your answer.

(c) Graph on the same graph the following functions (you may use other colors): $g(x) = 2f(x)$, $h(x) = f(x) + 4$, $s(x) = f(x-1)$ and $k(x) = -f(x)$.

a) $f(0) = 0$ $\qquad f(3) = 1$ $\qquad f(-1) = 1$

b) Symmetry w.r.t. y-axis $\Rightarrow$ EVEN



FOR g: