Test (diagnostic) MAT 1341A Fall 2016 4PM

1. A Cartesian equation of the plane which contains the point (3, 4, 2) and which is perpendicular to the planes $-x+2y+z=1$ and $-4y+3z=2$ is:

A. $10x-3y+4z=-50$
B. $-10x+3y+4z=50$
C. $10x-3y+4z=50$
D. $10x+3y-4z=50$
E. $10x+3y+4z=50$
F. $10x+3y+4z=-50$

These last 2 planes will intersect in a line with direction vector parallel to a normal n for the desired plane. So we compute:

$$n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 0 & -4 & 3 \end{vmatrix} = (10, +3, 4).$$ Since $(3,4,2)$ belongs to this plane, and $10.3 + 3.4 + 4.2 = 50$, E is correct.

2. An equation of the plane passing through the points $\overset{A}{(1, 2, -1)}$ and $\overset{B}{(2, 3, 1)}$ and parallel to the x-axis is:

A. $x+y-z=4$
B. $2x-z=-5$
C. $-x+2y=5$
D. $2y-z=5$
E. $2y+z=5$
F. $2x-z=5$

Such a plane will have a normal parallel to (1,0,0) $\times$ (B−A)

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-2,1)$$

Hence D is correct (as you check)

3. An equation of the plane containing the point $(-1, 1, 2)$ and the line with parametric equations $x = -1 + 2t,\ y = 4,\ z = 2 + t$ is:

A. $x + y - 2z = -5$

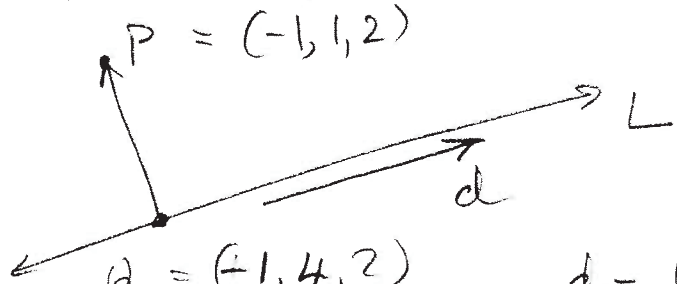
B. $x - 2z = -5$

C. $x + 2z = -5$

D. $x - 2z = 5$

E. $x + y + 2z = 5$

F. $x + 2z = 5$



$Q = (-1, 4, 2)$ $(t = 0)$ $\quad d = (2, 0, 1)$

$P - Q = (0, -3, 0)$

Such a plane will have a normal parallel to

$$(P-Q) \times d = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -3 & 0 \\ 2 & 0 & 1 \end{vmatrix} = (-3, -0, 6)$$

Hence $(1, 0, -2)$ is a normal. Since P belongs to this plane, and $1(-1) - 2(2) = -5$, an equation for this plane is $X - 2Z = -5$.

4. Parametric equations for the line containing $\overset{A}{(-1,\ 3,\ 4)}$ and $\overset{B}{(5,\ -1,\ 1)}$ are:

A. Such a line does not exist.

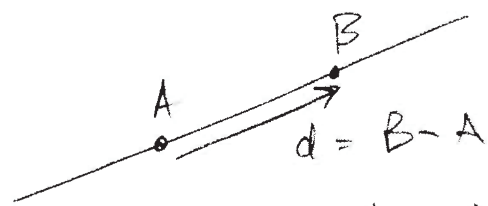
B. $x = -5 + 4t,\ y = 1 - 2t,\ z = 1;\ t \in \mathbf{R}$.

C. $x = 5 - 6t,\ y = -1 - t,\ z = 1 + 3t;\ t \in \mathbf{R}$.

D. $x = -1 - 6t,\ y = 3 + 4t,\ z = 4 + t;\ t \in \mathbf{R}$.

E. $x = -1 - 6t,\ y = 3 + 4t,\ z = 4 + 3t;\ t \in \mathbf{R}$.

F. $x = 5 + 6t,\ y = -1 + 4t,\ z = 2 + 3t;\ t \in \mathbf{R}$.



A direction vector for this line is parallel to $B - A = (6, -4, -3)$. Hence $(-6, 4, 3)$ is a direction vector and the only line above with this direction is E, which you can check is correct (as it contains A $(t = 0)$).

5. Find a Cartesian (scalar) equation for the plane with vector parametric equation

v_1 v_2

$$v = (0, 0, -2) + s(1, 1, 2) + t(-4, 2, 1);\ s, t \in \mathbf{R}.$$

A normal for this plane will be parallel to $v_1 \times v_2$ = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 2 \\ -4 & 2 & 1 \end{vmatrix}$

A. $x + y + 2z = -4$

B. $-4x + 2y + z = -2$

C. $x + 3y - 2z = 4$

D. $-x + 3y - 2z = -4$

E. $2x + 9y + 5z = -6$

F. $-2x + 9y + 5z = -1$

$= (-3, -9, 6) = -3(1, 3, -2)$

The only response with a normal parallel to this is C (which does indeed contain the point $(0, 0 -2)$).

S

v_1 v_2

6. The set of all vectors in $\mathbf{R}^3$ which are perpendicular to both $(1, -1, 5)$ and $(1, 2, 2)$ is:

A. $\{(-12,\ 3,\ 3)\}$

B. $\{(-12,\ t+3,\ t+3) \mid t \in \mathbf{R}\}$

C. $\{(0,\ -t,\ t) \mid t \in \mathbf{R}\}$

D. $\{(-4t,\ t,\ t) \mid t \in \mathbf{R}\}$

E. $\{(0,\ 0,\ 0)\}$

F. $\{(12,\ 3,\ 3)\}$

The set S will be the line through O in $\mathbb{R}^3$ with direction parallel to $v_1 \times v_2$ = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 5 \\ 1 & 2 & 2 \end{vmatrix}$

$= (-12, +3, 4) = 3(-4, 1, 1)$.

Hence, $S = \{t(-4, 1, 1) \mid t \in \mathbb{R}\}$

7. The angle θ between the vectors $(3,\ 0,\ -3)$ u and $(2,\ -2,\ -1)$ v is: determined

A. $\pi/7$
B. $\pi/6$
C. $\pi/5$
D. $\pi/4$
E. $\pi/3$
F. $\pi/2$

by the conditions 1) $0 \le \theta \le \pi$

2) $\cos\theta = \frac{u \cdot v}{\|u\|\,\|v\|} = \frac{9}{3\sqrt{2} \cdot 3}$

$= \frac{\sqrt{2}}{2}$

Hence $\theta = \pi/4$

8. The orthogonal projection $\mathrm{proj}_u\, v$ of $v = (1,\ -5,\ 8)$ along (or onto) $u = (0,\ 3,\ 3)$ is:

A. $(-\frac{1}{5},\ 1,\ -\frac{8}{5})$
B. $(\frac{1}{5},\ -1,\ \frac{8}{5})$
C. $(0,\ \frac{3}{2},\ \frac{3}{2})$
D. $(0,\ -\frac{3}{2},\ -\frac{3}{2})$
E. $(-1,\ 5,\ -8)$
F. $(1,\ -5,\ -8)$

given by

$\mathrm{proj}_u v = \left(\frac{v \cdot u}{\|u\|^2}\right) u$

$= \frac{9}{18} \cdot (0,3,3)$

$= (0, 3/2, 3/2)$

9. The volume of the parallelepiped with a vertex at the origin and edges given by the vectors $u = (2, 2, 4)$, $v = (2, 0, 5)$ and $w = (0, 1, 1)$ is:

A. 3
B. 7
C. 6
D. 10
E. 11
F. 14

This volume is $|u \times v \cdot w|$.

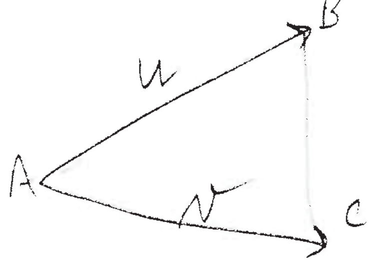
Now $u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 4 \\ 2 & 0 & 5 \end{vmatrix} = (10, -2, -4)$

Hence $u \times v \cdot w = (10, -2, -4) \cdot (0, 1, 1) = -6$

Thus vol $= 6$.

10. Find the area of the triangle with vertices $A = (-1, 4, 0)$, $B = (1, 3, 2)$ and $C = (1, 2, 0)$.

A. 1
B. 2
C. 3
D. 4
E. 5
F. 6



Area $= \frac{1}{2} \| u \times v \|$

$u = B - A = (2, -1, 2)$

$v = C - A = (2, -2, 0)$.

But $u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 2 & -2 & 0 \end{vmatrix} = (4, +4, -2)$
$= 2(2, 2, -1)$

Hence $\| u \times v \| = 2 \cdot \sqrt{9} = 6$. Hence the area is 3

11. Find

$$\left|\frac{6-2i}{1-2i}\right|. = \frac{2.|3-i|}{|1-2i|} = \frac{2\sqrt{10}}{\sqrt{5}} = 2\sqrt{2}$$

A. $1/2$
B. $\sqrt{2}/2$
C. $\sqrt{2}$
D. $2\sqrt{2}$
E. $3/2$
F. $\sqrt{14/11}$

12. The polar form of

$z_1 = r_1 e^{i\theta_1}$

$$\frac{z_1}{z_2} = \frac{3\sqrt{3}-3i}{\sqrt{2}+i\sqrt{2}}.$$

$z_2 = r_2 e^{i\theta_2}$

is:

A. $6(\cos(\frac{\pi}{12}) - i\sin(\frac{\pi}{12})) = 6e^{-i\frac{\pi}{12}}$
B. $3(\cos(\frac{\pi}{12}) - i\sin(\frac{\pi}{12})) = 3e^{-i\frac{\pi}{12}}$
C. $3(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})) = 3e^{i\frac{5\pi}{12}}$
D. $3(\cos(\frac{5\pi}{12}) - i\sin(\frac{5\pi}{12})) = 3e^{-i\frac{5\pi}{12}}$
E. $2(\cos(\frac{5\pi}{12}) - i\sin(\frac{5\pi}{12})) = 2e^{-i\frac{5\pi}{12}}$
F. $2(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12})) = 2e^{-i\frac{\pi}{12}}$

$r_1 = |z_1| = 3.\sqrt{4} = 6$

$\cos\theta_1 = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\sin\theta_1 = \frac{-3}{6} = -\frac{1}{2}$

$\therefore \theta_1 = -\pi/6$

$r_2 = |z_2| = \sqrt{2}.\sqrt{2} = 2$

$\cos\theta_2 = \frac{\sqrt{2}}{2}$

$\sin\theta_2 = \frac{\sqrt{2}}{2}$

$\therefore \theta_2 = \pi/4$

Hence $\frac{z_1}{z_2} = \frac{6e^{-i\pi/6}}{2e^{i\pi/4}} = 3.\ e^{i\pi(-1/6-1/4)} = \underline{3e^{-i\frac{5\pi}{12}}}$