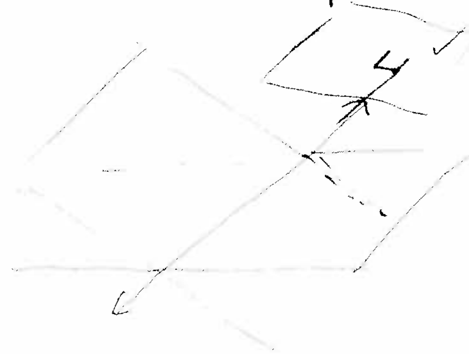
Test 1 (diagnostic) MAT 1341A Fall 2016 2:30PM.

1. A Cartesian equation of the plane which contains the point (2, 4, 3) and which is perpendicular to the planes with Cartesian equations $x + 2y - z = 1$ and $3x - 4y = 2$ is:

A. $4x - 3y + 10z = -50$
B. $4x + 3y - 10z = 50$
C. $4x - 3y + 10z = 50$
D. $-4x + 3y + 10z = 50$
E. $4x + 3y + 10z = -50$
[✓ F.] $4x + 3y + 10z = 50$

These last two plane will intersect in a line with direction vector that will serve as a normal for the desired plane so we compute



$$n = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -4 & 0 \end{vmatrix} = (-4, -3, -10)$$

or $(4, 3, 10)$.

Since $(2,4,3)$ belongs to plane, and

$$4.2 + 3.4 + 3.10 = 50,$$

an equation for the plane is given by

$$4x + 3y + 10z = 50$$

2. An equation of the plane passing through the points $\overset{A}{(2,\ 1,\ -1)}$ and $\overset{B}{(3,\ 2,\ 1)}$ and parallel to the y-axis is:

A. $x + y - z = 4$
B. $2y - z = 5$
C. $2x - y = 5$
[✓ D.] $2x - z = 5$
E. $2x + z = 5$
F. $2y - z = -5$

Such a plane will have as a normal

$$n = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = (2, 0, -1) \quad \left(\begin{array}{c} B - A = \\ (1, 1, 2) \end{array} \right)$$

Since there is only one plane in the responses with normal $(2, 0, -1)$, ⓓ must be correct (as you can check)

P L

3. An equation of the plane containing the point $(1, -1, 2)$ and the line with parametric equations $x = 4,\ y = -1 + 2t,\ z = 2 + t$ is:

A. $x + y - 2z = -5$

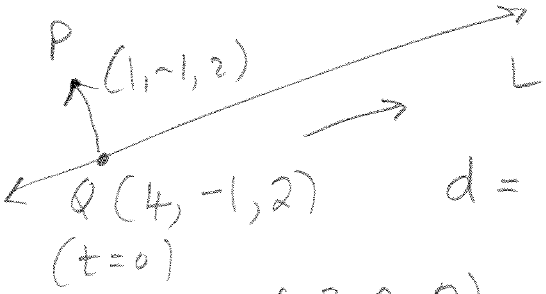
[✓ B.] $y - 2z = -5$

C. $y + 2z = -5$

D. $y - 2z = 5$

E. $x + y + 2z = 5$

F. $y + 2z = 5$



$d = (0, 2, 1)$

$P - Q = (-3, 0, 0)$

Such a plane will have normal parallel to

$$(P-Q) \times d = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & 0 \\ 0 & 2 & 1 \end{vmatrix} = (0, +3, -6)$$

Hence we may take $(0, 1, -2)$ as normal. Since P belongs to this plane, and $1(-1) - 2(2) = -5$, an equation for this plane is $y - 2z = -5$.

P Q

4. Parametric equations for the line containing $(3,\ -1,\ 4)$ and $(-1,\ 5,\ 1)$ are:

A. Such a line does not exist.

B. $x = 1 - 2t,\ y = -5 + 4t,\ z = 1;\ t \in \mathbf{R}$.

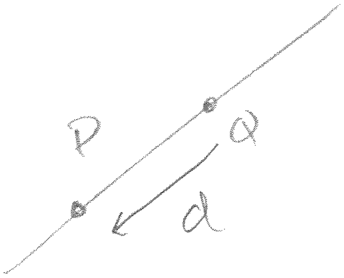
C. $x = -1 - t,\ y = 5 - 6t,\ z = 1 + 3t;\ t \in \mathbf{R}$.

D. $x = 3 + 4t,\ y = -1 - 6t,\ z = 4 + t;\ t \in \mathbf{R}$.

[✓ E.] $x = 3 + 4t,\ y = -1 - 6t,\ z = 4 + 3t;\ t \in \mathbf{R}$.

F. $x = -1 + 4t,\ y = 5 + 6t,\ z = 2 + 3t;\ t \in \mathbf{R}$.

A direction vector for this line is $P - Q = (4, -6, 3)$. The only line in the list with this direction vector is Ⓔ (and it contains P $(t=0)$).



5. Find a Cartesian (scalar) equation for the plane with vector parametric equation

$$v = (0,0,-2) + s\overset{v_1}{(1,1,2)} + t\overset{v_2}{(2,-4,1)};\ s,t \in \mathbf{R}.$$

A. $x + y + 2z = -4$

B. $2x - 4y + z = -2$

[✓ C.] $3x + y - 2z = 4$

D. $3x - y - 2z = -4$

E. $9x + 2y + 5z = -6$

F. $9x - 2y + 5z = -1$

A normal for this plane will be parallel to $v_1 \times v_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & -4 & 1 \end{vmatrix}$

$= (9, +3, -6)$

Hence $(3, 1, -2)$ is a normal. The only response with a normal parallel to this is (C), (which does indeed contain $(0,0,-2)$).

6. The set S of all vectors in $\mathbf{R}^3$ which are perpendicular to both $\overset{v_1}{(-1,1,5)}$ and $\overset{v_2}{(2,1,2)}$.

A. $\{(3,\ -12,\ 3)\}$

B. $\{(t+3,\ -12,\ t+3) \mid t \in \mathbf{R}\}$

[✓ C.] $\{(t,\ -4t,\ t) \mid t \in \mathbf{R}\}$

D. $\{(-t,\ 0,\ t) \mid t \in \mathbf{R}\}$

E. $\{(0,\ 0,\ 0)\}$

F. $\{(3,\ 12,\ 3)\}$

This set will be the line through 0 in $\mathbb{R}^3$ with direction $\parallel$ to $v_1 \times v_2$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 5 \\ 2 & 1 & 2 \end{vmatrix} = (-3, +12, -3)$

Hence $(1, -4, 1)$ is a dirⁿ vector for this line so

$S = \{t(1,-4,1) \mid t \in \mathbb{R}$

7. The angle θ between the vectors $(0,\ \overset{u}{3},\ -3)$ and $(-2,\ \overset{v}{2},\ -1)$ is:

- A. $\pi/6$
- B. $\pi/2$
- [✓ C. $\pi/4$]
- D. $\pi/3$
- E. $\pi/5$
- F. $\pi/7$

The angle θ satisfies $\cos\theta = \dfrac{u\cdot v}{\|u\|\,\|v\|}$,

and $0 \le \theta \le \pi$. Hence $\cos\theta = \dfrac{9}{3\sqrt{2}\cdot 3} = \dfrac{\sqrt{2}}{2}$.

Hence $\theta = \pi/4$

8. The orthogonal projection $\text{proj}_u\, v$ of $v = (-5,\ 1,\ 8)$ along $u = (3,\ 0,\ 3)$ is: given by

- A. $(1,\ -\frac{1}{5},\ -\frac{8}{5})$
- B. $(-1,\ \frac{1}{5},\ \frac{8}{5})$
- C. $(-\frac{3}{2},\ 0,\ -\frac{3}{2})$
- [✓ D. $(\frac{3}{2},\ 0,\ \frac{3}{2})$]
- E. $(5,\ -1,\ -8)$
- F. $(-5,\ 1,\ -8)$

$$\text{proj}_u v = \frac{v\cdot u}{\|u\|^2}\, u$$

$$= \frac{9}{18}(3, 0, 3)$$

$$= (3/2, 0, 3/2)$$

9. The volume of the parallelepiped with a vertex at the origin and edges given by the vectors $u = (1, 1, 2)$, $v = (0, 2, 5)$ and $w = (1, 0, 1)$ is:

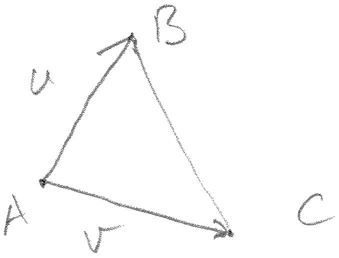
[✓ A. 3]
B. 7
C. 9
D. 10
E. 11
F. 14

Now, vol = absolute value of $u \times v \cdot w$.

Now $u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 0 & 2 & 5 \end{vmatrix} = (1, -5, 2)$

Hence $u \times v \cdot w = (1, -5, 2) \cdot (1, 0, 1) = 3$.

10. Find the area of the triangle with vertices $A = (-1,\ 5,\ 0)$, $B = (1,\ 0,\ 4)$ and $C = (1,\ 4,\ 0)$.



area $= \frac{1}{2} \|u \times v\|$,
$u = B - A = (2, -5, 4)$,
$v = C - A = (2, -1, 0)$.

A. 1
B. 2
C. 3
D. 4
E. 5
[✓ F. 6]

But $u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 4 \\ 2 & -1 & 0 \end{vmatrix} = (4, +8, 8) = 4(1, 2, 2)$

Hence $\|u \times v\| = 4 \cdot \sqrt{9} = 12$. Hence the area is 6.

11. Find

$$\left|\frac{3-i}{2-4i}\right|. = \frac{|3-i|}{|2-4i|} = \frac{\sqrt{10}}{\sqrt{20}}$$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

A. 1/2
[✓ B. $\sqrt{2}/2$]
C. $\sqrt{8/11}$
D. 8/25
E. 3/2
F. $\sqrt{14/11}$

12. Find the polar form of

$$z_1 = r_1 e^{i\theta_1}$$

$$\frac{z_1}{z_2} = \frac{5+5\sqrt{3}i}{\sqrt{2}-\sqrt{2}i.}$$

$$z_2 = r_2 e^{i\theta_2}$$

$r_1 = |z_1| = 5\sqrt{4} = 10$

$\cos\theta_1 = \frac{5}{10} = \frac{1}{2}$

$\sin\theta_1 = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

Hence $\theta_1 = \pi/3$

$r_2 = |z_2| = \sqrt{2}.\sqrt{2} = 2$

$\cos\theta_2 = \frac{\sqrt{2}}{2}$

$\sin\theta_2 = -\frac{\sqrt{2}}{2}$

$\therefore \theta_2 = -\pi/4$

A. $5(\cos(\frac{5\pi}{12}) - i\sin(\frac{5\pi}{12})) = 5e^{-i\frac{5\pi}{12}}$
B. $5(\cos(\frac{11\pi}{12}) + i\sin(\frac{11\pi}{12})) = 5e^{i\frac{11\pi}{12}}$
C. $5(\cos(\frac{7\pi}{12}) - i\sin(\frac{7\pi}{12})) = 5e^{-i\frac{7\pi}{12}}$
D. $5(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})) = 5e^{i\frac{5\pi}{12}}$
E. $5(\cos(\frac{11\pi}{12}) - i\sin(\frac{11\pi}{12})) = 5e^{-i\frac{11\pi}{12}}$
[✓ F. $5(\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12})) = 5e^{i\frac{7\pi}{12}}$]

Hence $\frac{z_1}{z_2} = \frac{10e^{i\pi/3}}{2e^{-i\pi/4}} = 5\, e^{i\pi(\frac{1}{3}+\frac{1}{4})} = 5e^{i\frac{7\pi}{12}}$