# MAT 1341A Fall 2016 Final Exam Marking Scheme

**11.** (Network of streets)

a) Write down a system of linear equations which describes the the traffic flow, together with all the constraints on the v
$x_i$, $i = 1, \ldots, 5$.

**[Points: 3= 2 if all 5 equations correct; less .5 for each equation incorrect (min 0) plus 1=2*.5 for the two constraints ]**

b) The reduced row-echelon form of the augmented matrix of the system in part (a) is ...
Give the general solution. (Ignore the constraints from (a) at this point.)

**[Points: 2= 2 for all 5 components correct; less .5 for each components incorrect (min 0) ]**

c) Find the maximum and minimum flow along BC, **using your results from (b).**
*(You must justify all your answers.)*

**[Points: 1= 2*.5 each correct answer ]**

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**12.** Let $A = \ldots$

**[Points: .5= correct RRE form ]**

a) Find a basis for the column space $\mathrm{col}\,(A)$ of $A$.

**[Points: 1.5 = .5 knowing what to do + 1 consistent with RRE form ]**

b) Find a basis for $\ker(A)$.

**[Points: 1.5 = .5 knowing what to do + .5 general solution consistent with RRE+ .5 any basis consistent with their general solution. ]**

c) Give a complete geometric description of $\ker(A)$.

**[Points: 1.5 = .5 'plane/line' + + .5 point + .5 normal/dirction vectors, consistent with (b) ]**

d) Extend your basis of $\mathrm{col}\,(A)$ to a basis of $\mathbf{R}^4$.

**[Points: 1= .5 basis of $\mathbf{R}^4$, +.5 justified answer consistent with (a) ]**

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**13.** Let $U = \{(x, y, z, w) \in \mathbf{R}^4 \mid x + z + w = 0\}$.

a) Find a basis of $U$ and give the dimension of $U$.

**[Points: 2 = .5 correct general solution + .5 any basis conistent with their general soltuion +.5 dimsnsion consistent with their basis + .5 justified – i.e. they must show some work; as they have the 'basis for the kernel' algorithm, they don't have to prove it spans and is l.i.. ]**

b) Find an orthogonal basis of $U$.

**[Points: 2 = 1 knowing G-S (correctly) + .5 all vectors othogonal + .5 all vectors in U ]**

c) Find the best approximation by a vector in $U$ to the vector $(1, 1, 1, 0)$.

**[Points: 2= .5 knowing to compute the projection + .5 correct projection formula + .5 a (non-zero) vector in $W$ + an actual .5 correct answer ]**

**14.** Let $A = ....$

a) Find the characteristic polynomial of $A$, and factor it to show that the only eigenvalues of $A$ are 1 and 2.

**[Points: 1= .5 correct characteristic polynomial + .5 factored with sufficient justification ]**

b) Find a basis of $E_1$...

**[Points: 1= .5 correct RRE form of $A - I$,+ .5 (non-zero) vector consistent with their general solution/RRE form ]**

c) Find a basis of $E_2$...

**[Points: 2= 1 correct RRE form of $A - 2I$ + .5 for each (non-zero) vector consistent with their general solution/RRE form ]**

d) Find an invertible matrix $P$ such that $P^{-1}AP = D$ is diagonal, and give this diagonal matrix $D$. Explain why your choice of $P$ is invertible.

**[Points: 2 = 1 assembling a (a 3 by 3, no zero columns) $P$ consistent with their answers to (b) and (c) (only .5 is they give $P^t$) + .5 $D$ consistent with their $P$ + .5 justification of P's invertibility (theorem or computation) + .5 if their answer is actually correct; If they conclude (consistent with their answers to (c) and (d)) that no such P exists, max 1pt. ]**

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**15 (a).** State whether each of the following is (always) true, or is (possibly) false, in the box after the statement.

**[Points: Each part : 1.5 =.5 (correct) + 1 (justification) ]**

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**16.**

**[Points: 3= 1.5 * 2 parts ]**