

# MAT 1341A — Test 4 MARKING SCHEME (draft)

3. Let

$$A = \dots$$

(a) Find a basis for the row space of  $A$ .

[2= 1 correct “RE/RRE” form + 1 answer consistent with their “RE/RRE” form]

(b) Find a basis for the column space of  $A$ .

[1.5= 1 correct method + .5 answer consistent with their “RE/RRE” form]

(c) Find a basis for  $\ker A = \{x \in \mathbf{R}^4 \mid Ax = 0\}$ .

[1.5= .5 correct method + .5 answer consistent with their “RE/RRE” form + .5 correct answer]

(d) Find the dimension of  $\{Ax \mid x \in \mathbf{R}^4\}$ .

[1= .5 knowing what this subspace is + .5 correct answer]

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4. Let  $u_1 = (0, 1, 1, 0)$ ,  $u_2 = (0, 0, 0, 1)$ ,  $u_3 = (0, 1, -1, 0)$ , and  $U = \text{span}\{u_1, u_2, u_3\}$ .

a) Show that  $\{u_1, u_2, u_3\}$  is an orthogonal set.

[1 = .5 checking the 3 dot products + .5 ”vectors are not zero”]

b) Briefly explain why  $\{u_1, u_2, u_3\}$  is a basis of  $U$ .

[1 = 1 ”orthogonal sets are l.i.”]

c) Find the best approximation to  $(1, -1, 2, -1)$  by vectors in  $U$ .

[3= 1 knowing to compute the projection + 1 correct projection formula + .5 a (non-zero) vector in  $U$  + .5 correct answer]

d) Extend  $\{u_1, u_2, u_3\}$  to a basis of  $\mathbf{R}^4$ .

[1= .5 any correct extension + .5 justification]

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5(a). State whether the following is true or false, and justify your answer.

[3 = 2\*(.5 correct answer + 1 justification)]

5(b). Let  $A$  be a  $n \times n$  matrix with real entries. Give three additional statements equivalent to

“ $A$  is invertible”

[3= 3 \* 1 point each]

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6. [Bonus] Suppose  $A$  is an invertible  $15 \times 15$  matrix and  $B$  is any  $15 \times 12$  matrix with  $\text{rank } B = 12$ . Prove carefully that  $\text{rank } AB = 12$ .

[3= as on Test 3]