MAT 1341A –Marking Scheme– Test 3, 2016

This is for one version one but can and should be used for each version.

3. Suppose $p, q \in \mathbf{R}$ and consider the linear system in x, y and z:

$$\begin{array}{rcrcrcl} x & & & - & z & = & 2 \\ -x & + & y & + & z & = & p \\ x & + & 2y & + & pz & = & 2p+q \end{array}$$

a) If $[\,A\,|\,b\,]$ is the augmented matrix of the system above, find $\operatorname{rank} A$ and $\operatorname{rank}[\,A\,|\,b\,]$ for <u>all</u> values of p and q.

Marks: [3 = 1 correct RRE form + 1 rank A + 1 rank$[A|b]$; the last two answers need only be consistent with their RRE form, but if an error results in a considerably simpler problem, give only 1.5 points in total]

4b). Using part (a), find all values of p and q so that this system has

(i) a unique solution,

Marks: [.5 = .5 consistent with (a)]

(ii) infinitely many solutions, or

Marks: [.5 = .5 consistent with (a)]

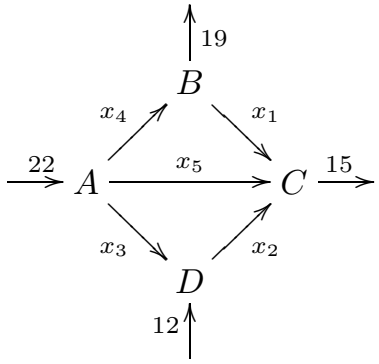
(iii) no solutions.

Marks: [.5 = .5 consistent with (a)]

4c). In case b(ii) above, give a complete geometric description of the set of solutions.

Marks: [1.5= .5 for a general solution consistent with their RRE form in (a) + 1 point for a response consistent with their general solution (this is a 3-part description: one -part only correct yields 0/1, two parts correct .5/1, three parts correct 1/1]

4. Consider the network of streets ...



a) Write down a system of linear equations which describes the traffic flow, together with all the constraints on the variables x_i, $i = 1, \ldots, 5$.

Marks: [2 = 1 − .5 × (number of incorrect equations), minimum 0; + .5 " each x_i is an integer" + .5 " each x_i is non-negative"]

b) The reduced row-echelon form of the augmented matrix of the system in part (a) is

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & 0 & -19 \\ 0 & 1 & 0 & 1 & 1 & 34 \\ 0 & 0 & 1 & 1 & 1 & 22 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

Marks: [2 = 2 − .5× (number of incorrect equations), minimum 0]

c) If $\overline{AC}$ were closed due to roadwork, **using your results from (b) and the constraints**, find

(i) The maximum flow along $\overline{DC}$, and
(ii) The minimum flow along $\overline{DC}$.

Marks: [2= 2*(.5 correct +.5 justification)]

5. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers, matrices, or functions (as is appropriate), if possible, or an argument using theorems and facts from class.

- If you say the statement is always true, you must give a clear explanation.

Marks: Each part: 1.5 = .5 (correct) + 1 correct justification/ counterexample

6. [Challenge/Bonus] If A and B are 3 by 3 matrices with $B \neq 0$ and $AB = 0$, then rank $A < 3$.

Marks: [3 = 1 for some correct and useful idea + 1 for some good progress + 1 if all is correct and well-written. No ".5"s here.]