New examples of W* and C*-superrigid groups

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Summer School in Operator Algebras - Fields Institute and University of Ottawa

Group von Neumann algebras

Murray and von Neumann

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Definition. The **reduced C*-algebra of** Γ, denoted by $C^*_r(\Gamma)$, is the norm closure of $\mathbb{C}[\Gamma] \subset \mathbb{B}(\ell^2(\Gamma))$.
$\rightsquigarrow \mathbb{C}[\Gamma] \subset C^*_r(\Gamma) \subset L(\Gamma)$.

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- Can Γ be completely recovered from $L(\Gamma)$?

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Dykema '93. If $\Gamma_1, \ldots, \Gamma_n$ and $\Lambda_1, \ldots, \Lambda_n$ are infinite amenable groups, then $L(\Gamma_1 * \cdots * \Gamma_n) \cong L(\Lambda_1 * \cdots * \Lambda_n)$.

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Popa's strong rigidity theorem '04: If $G_i = \mathbb{Z}_2 \wr \Gamma_i$, where Γ_i is a property (T) group for any i with $L(G_1) \cong L(G_2)$, then $G_1 \cong G_2$.

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Conjecture (Connes '80s, Popa '07)

Any icc property (T) group is W*-superrigid.

$\rightsquigarrow$ Completely open.

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Recall. If $\Gamma \curvearrowright I$, then the *generalized wreath product group* $\Sigma \wr_I \Gamma$ is $\Sigma^{(I)} \rtimes_\sigma \Gamma$, where $\sigma_g((x_i)_{i\in I}) = (x_{g^{-1}i})_{i\in I}$.

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Theorem (Ioana, Popa, Vaes '10)

Let $G = \mathbb{Z}_2 \wr_{K/B} K$, where K is icc property (T) group and $B < K$ is infinite amenable malnormal (i.e. $gBg^{-1} \cap B$ is finite for any $g \in K \setminus B$).
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$\rightsquigarrow$ $C_r^*(G)$ completely remembers G.

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(G has trivial amenable radical, so $C_r^*(\Gamma)$ has a unique trace by Breuillard, Kalantar, Kennedy, and Ozawa '14)

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Remark (Phillips '87). If $L(\Lambda)$ is a full factor, then there exist uncountably many unitaries $w \in L(\Lambda)$ that implement outer automorphisms of $C_r^*(\Lambda)$.

Corollary (Chifan, Ioana '17)

Let $G = (K \times K) *_{\Delta(B)} (K \times K)$, where $B = \mathbb{Z} \wr \mathbb{Z}, K = \mathbb{Z} \wr \mathbb{F}_n$ and $\Delta(B) = \{(b,b) | b \in B\} < K \times K$.
Then G is C*-superrigid.

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$\rightsquigarrow$ First class of C*-superrigid groups.

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$\rightsquigarrow$ Almost a stability result: If Γ_0 is a hyperbolic, property (T) group such that $A_0 \rtimes \Gamma_0$ is W*-superrigid satisfying "certain conditions", then $A_0^I \rtimes \tilde{\Gamma}$ is W*-superrigid.

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