Dimension and tensorial absorption in operator algebras

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Recurring theme

We shall compare the studies of the structure of C^*-algebras and of von Neumann algebras.

We will stick to the separable case.

Remark

There are not many abelian von Neumann algebras $L^\infty(X)$ but many abelian C^*-algebras $C_0(X)$.

v.N. setting: $X =$ (possibly) $[0,1]$ plus $\leq \aleph_0$ isolated points.

C^*-setting: $X =$ any 2^{nd} countable locally compact Hausdorff space, up to homeomorphism.

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Regularity: overview

Fundamental to the study of the structure of operator algebras are notions of regularity.

Classically, this is interpreted as **amenability**.

More recently (particularly, in the C^*-setting), **dimension** and **tensorial absorption** seem to be more relevant.

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Vastly generalizing partitions of unity in the commutative case, we have the following:

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C^*-setting (CPAP): "approx." means **point-norm**: for any finite subset $\mathcal{F} \subset A$ and any $\epsilon > 0$, $\exists$ (F, ϕ, ψ) s.t.

$$\|\phi\psi(x) - x\| < \epsilon.$$

v.N. case (semidiscrete): "approx." means **point-weak***: for any finite $\mathcal{F} \subset A$ and any weak* nbhd. V of 0, $\exists$ (F, ϕ, ψ) s.t.

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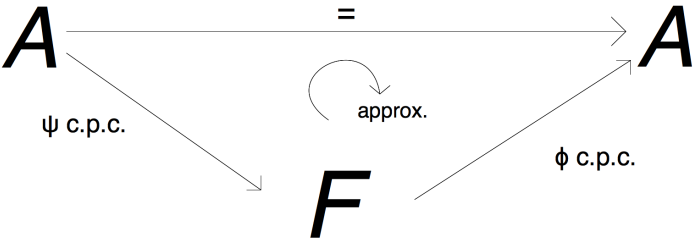

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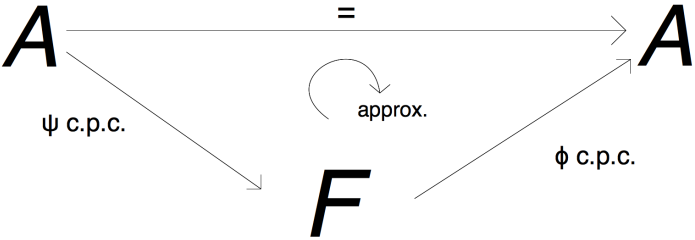

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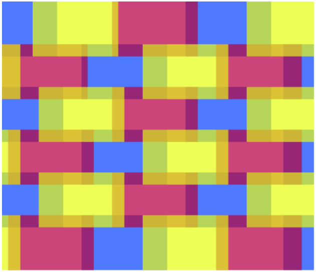

Covering dimension can be phrased using partitions of unity.

Proposition

Let X be a compact metric space. Then $\dim X \leq n$ iff for any open cover $\mathcal{U}$ of X, $\exists$ a partition of unity $(e_\alpha)_{\alpha \in A} \subset C(X)_+$ subordinate to $\mathcal{U}$ which is $(n+1)$-colourable:

$$A = A_0 \amalg \cdots \amalg A_n,$$

such that $(e_\alpha)_{\alpha \in A_i}$ are pairwise orthogonal.

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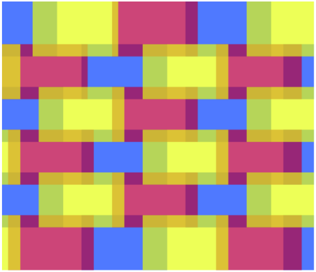

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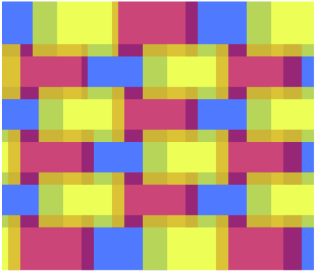

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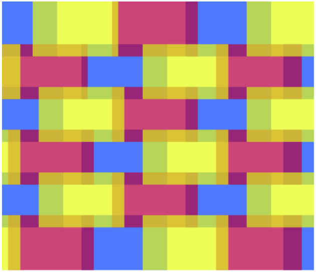

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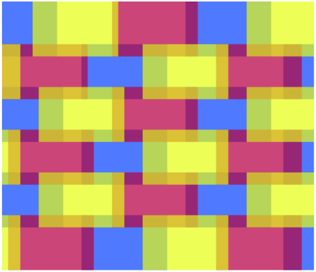

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A C^*-alg. A has decomposition rank $\leq n$ if

Order 0 means orthogonality preserving,
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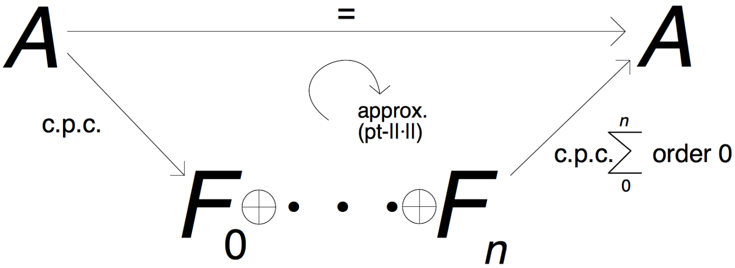

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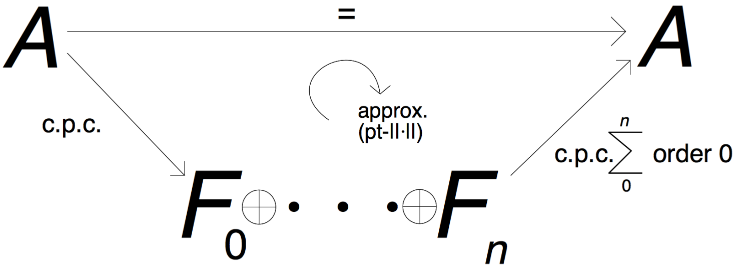

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$\dim_{nuc}(A) \leq \mathrm{dr}\,(A)$.

While $\mathrm{dr}\,(A) < \infty$ implies A is quasidiagonal, $\dim_{nuc}(\mathcal{O}_n) = 1$ (for $n < \infty$) for example.

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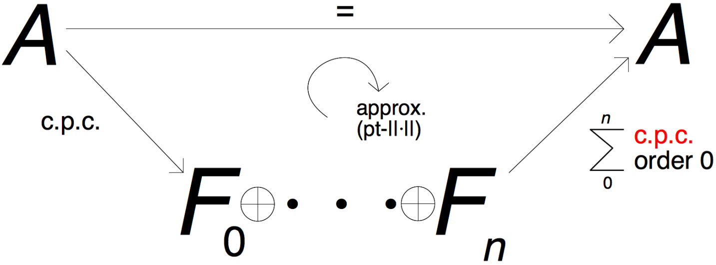

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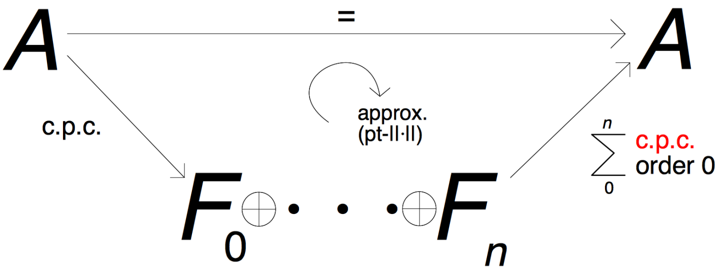

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Hirshberg-Kirchberg-White ('12): All amenable von Neumann algebras have semidiscreteness dimension 0.

Amenability and dimension, remarks

Theorem (Hirshberg-Kirchberg-White '12)

If A is an amenable C^*-algebra, then the map ϕ in the CPAP can always be taken to be n-colourable, for some n.

But, n depends on the degree of approximation (i.e. on $\epsilon > 0$ and the finite set $\mathcal{F} \subset A$); it may not be bounded.

Winter ('03): $\dim_{nuc} C(X) = \mathrm{dr}\, C(X) = \dim(X)$, so there exist amenable C^*-algebras with $\dim_{nuc}$ arbitrarily large, even ∞.

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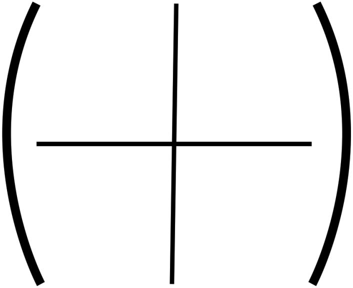

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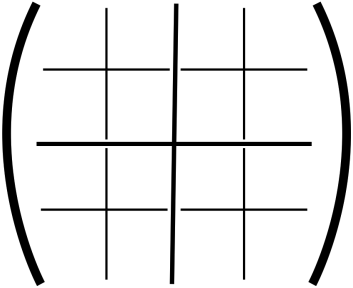

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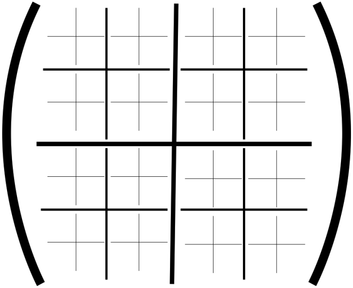

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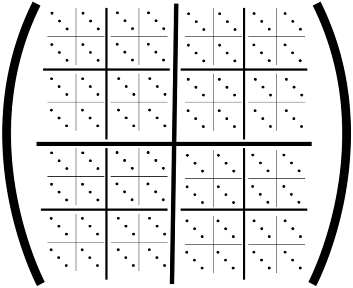

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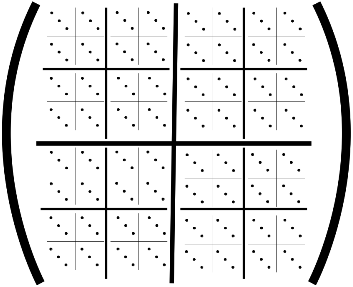

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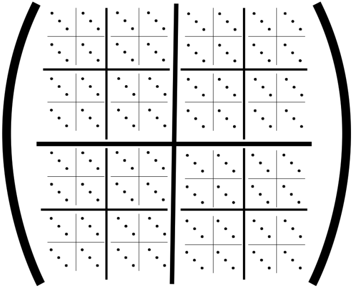

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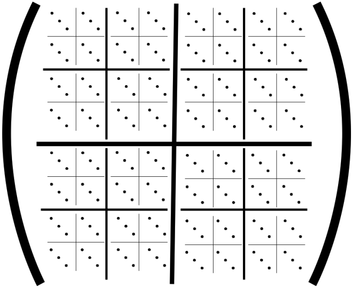

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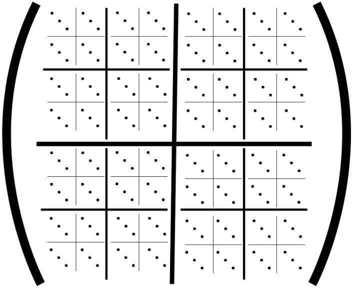

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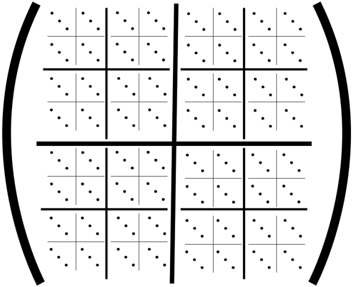

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Moreover, projections are rarely divisible in C^*-algebras, so we would be crazy to expect $A \cong A \otimes M_{p^\infty}$ to hold for all (or many) simple, amenable, non-type I C^*-algebras A.

The Jiang-Su algebra

The Jiang-Su algebra is like a UHF algebra, but no projections.

Construction:

More precisely, let p, q be coprime,

$$\mathcal{Z}_{p^\infty,q^\infty} := \{f \in C([0,1], M_{p^\infty} \otimes M_{q^\infty}) | f(0) \in M_{p^\infty} \otimes 1_{q^\infty}, \\ f(1) \in 1_{p^\infty} \otimes M_{q^\infty}\}.$$

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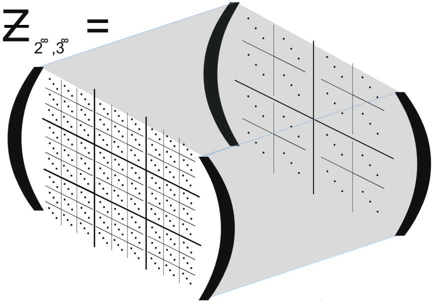

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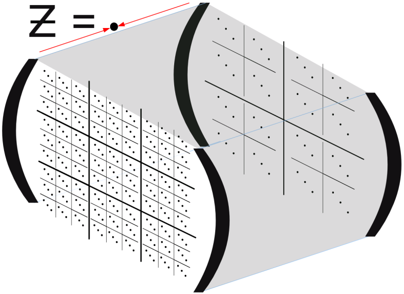

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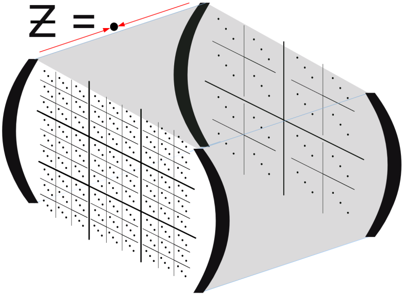

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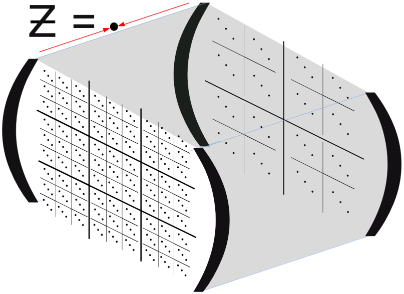

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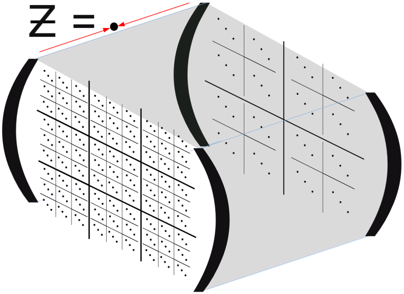

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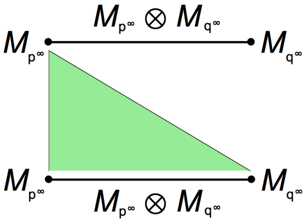

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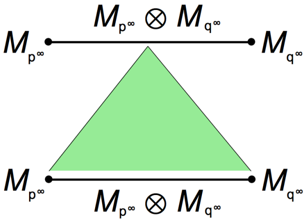

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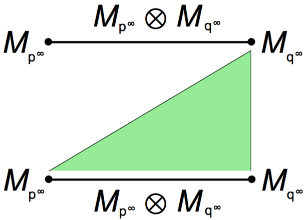

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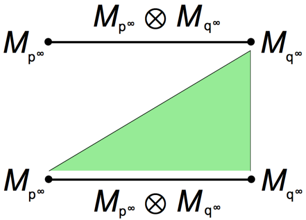

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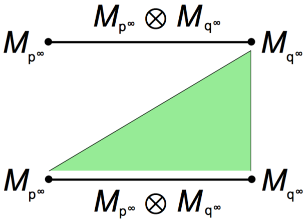

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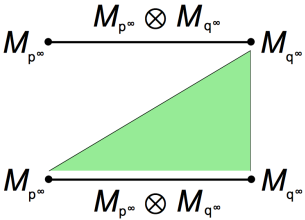

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Definition

A C^*-algebra (von Neumann algebra) A is $\mathcal{D}$-absorbing if $A \cong A \otimes \mathcal{D}$.

(Of course, the meaning of $\otimes$ is different in the C^*- and von Neumann cases.)

$\mathcal{D}$-absorption adds uniformity and regularity.

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$\mathcal{D}$-absorption implies $\mathcal{Z}$-absorption.

$\mathcal{Z}$-absorption does not come for free for amenable C^*-algebras: Villadsen's examples (with $\dim_{nuc} = \infty$) are not $\mathcal{Z}$-absorbing.

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$\mathcal{Z}$-absorption and dimension

Theorem (Winter '12, Robert '11, T '12)

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Conjecture (Toms-Winter)

For a simple, separable, amenable, non-type I C^*-algebra A, TFAE:

(i) $\dim_{nuc} A < \infty$;

(ii) A is $\mathcal{Z}$-absorbing.

One also expects to be able to classify the algebras satisfying these conditions which satisfy the Universal Coefficient Theorem, using K-theory and traces.

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v.N. algebras	**C^*-algebras**
Amenable $\Leftrightarrow \dim_{sd} = 0$	Amenable $\overset{\not\Rightarrow}{\Leftarrow} \dim_{nuc} < \infty$
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Dimension reduction (C^*-algebras)

Conjecture (Toms-Winter)

For a simple, separable, amenable C^*-algebra A,
(i) $\dim_{nuc} A < \infty \Leftrightarrow$ (ii) A is $\mathcal{Z}$-absorbing.

(ii) $\Rightarrow$ (i) is a matter of dimension reduction.
For many classes of C^*-algebras (such as simple AH algebras, i.e. inductive limits of certain homogeneous C^*-algebras), (ii) $\Rightarrow$ (i) is known through classification:

1. A class $\mathcal{C}$ of $\mathcal{Z}$-stable C^*-algebras is classified (by K-theory and traces);
2. The class $\mathcal{C}$ is shown to contain certain models which exhaust the invariant;
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Classification has only been shown with restrictions on the C^*-algebras in $\mathcal{C}$, such as a certain inductive limit structure (and simplicity).

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A significantly different approach to dimension reduction:

Theorem (Kirchberg-Rørdam '04)

For any space X, $\dim_{nuc} C_0(X, \mathcal{O}_2) \leq 3$.

The proof is short, and mostly uses $K_*(\mathcal{O}_2) = 0$ (more specifically, that the unitary group of $C(S^1, \mathcal{O}_2)$ is connected).

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Ideas in the proof:

Reduce to M_{p^∞} in place of $\mathcal{Z}$, using UHF fibres in $\mathcal{Z}_{p^\infty, q^\infty}$.

Want to use Kirchberg-Rørdam's result, requiring us to put $C_0(Y, \mathcal{O}_2)$ into $C_0(X, M_{p^\infty})$ somehow.

The cone over $\mathcal{O}_2$ is quasidiagonal, allowing us to approximately embed it into M_{p^∞}.

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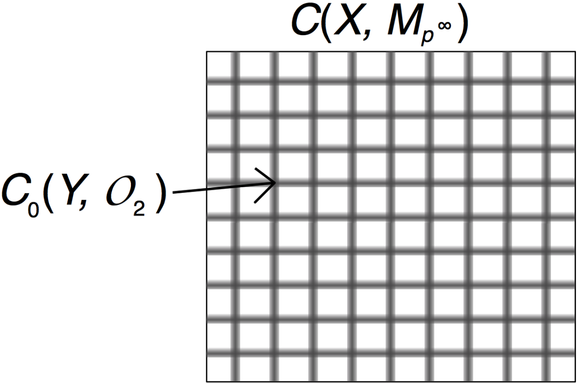

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Is it an inductive limit of subhomogeneous C^*-algebras with $\dim_{nuc} \leq 2$?

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Equivalently, is $\dim_{nuc}(A \otimes \mathcal{Z})$ universally bounded for such A?

Current project: extend our result to A subhomogeneous (hence even locally subhomogeneous).

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