Decomposition rank and Jiang-Su stability

Aaron Tikuisis
a.tikuisis@uni-muenster.de

University of Münster

Fields workshop on applications to operator algebras

The Toms-Winter conjecture

Fact (Toms, '08)

There exist 2 non-isomorphic simple, separable, unital, nuclear C^*-algebras with the same K-theory and traces.

Conjecture

For a simple, separable, unital, nonelementary, nuclear C^*-algebra A, the following are equivalent:

(i) A is $\mathcal{Z}$-stable;

(ii) A has finite nuclear dimension;

(iii) A has strict comparison of positive elements;

(iv) A is an inductive limit of nice building blocks (2-NCCW complexes, direct sums of $M_n \otimes \mathcal{O}_m \otimes C(\mathbb{T})$).

Moreover, the algebras satisfying (i)-(iv) are classifiable.

Note: exactly one of Toms' algebras satisfy (i)-(iv).

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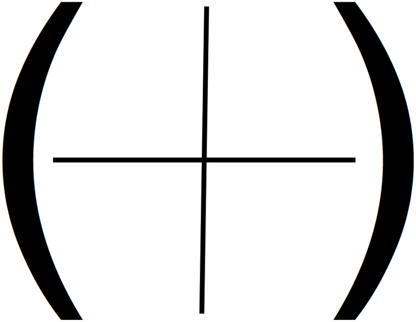
UHF algebras:



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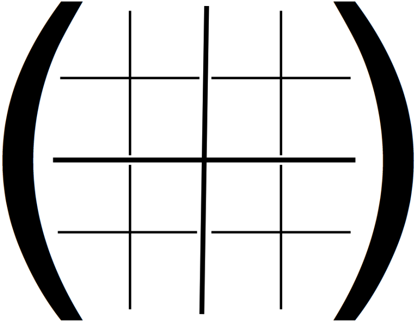

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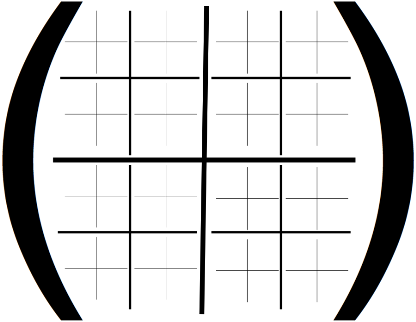

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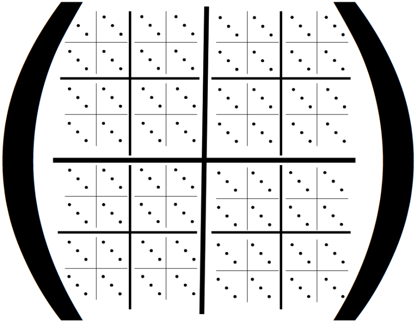

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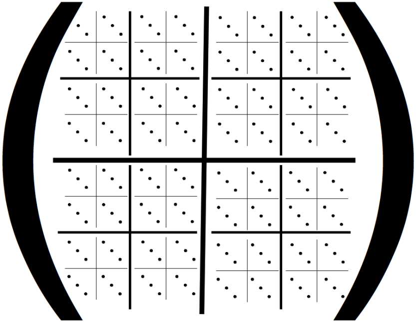
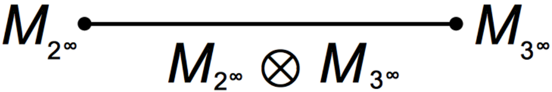


M_{2^∞}

M_{n^∞}-stable algebras (of the form $A \otimes M_{n^\infty}$) are very regular: UHF adds uniformity.

Jiang-Su algebra:



$\mathcal{Z}$ is a simple inductive limit of $\mathcal{Z}_{2^\infty,3^\infty}$ (pictured), with unique trace.

Like a UHF algebra, satisfies $\mathcal{Z} \cong \mathcal{Z} \otimes \mathcal{Z}$ and $\mathcal{Z}$-stability adds uniformity.

$K_*(\mathcal{Z}) = K_*(\mathbb{C})$, so $\mathcal{Z}$-stability is much less restrictive than UHF-stability.

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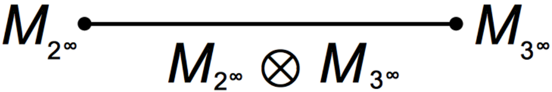

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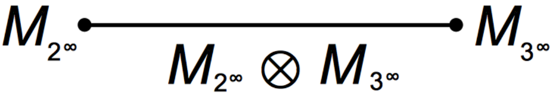

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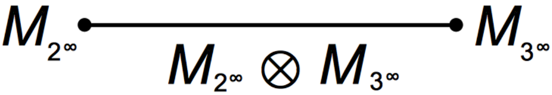

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Dimension

Covering dimension

$\dim X \leq n$ if and only if for every open cover $\mathcal{U}$ of X,
$\exists$ a partition of unity $\{e_\lambda\}_{\lambda \in \Lambda} \subset C(X, \mathbb{C})$ of nonnegative functions s.t.

(i) $\{e_\lambda\}_{\lambda \in \Lambda}$ is $(n+1)$-colourable, where functions $e_{\lambda_1}, e_{\lambda_2}$ of the same colour must be orthogonal, i.e. $e_{\lambda_1} e_{\lambda_2} = 0$; and

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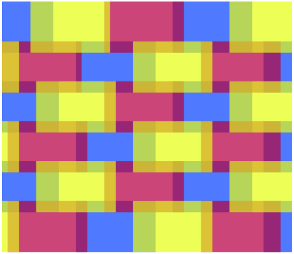

Decomposition rank (Kirchberg-Winter ’04)
A C^*-alg. A has decomposition rank $\leq n$ if

Order 0 means orthogonality preserving,
$ab = 0 \Rightarrow \phi(a)\phi(b) = 0$.

Think: (controlled) noncommutative span, $(n+1)$ colours.

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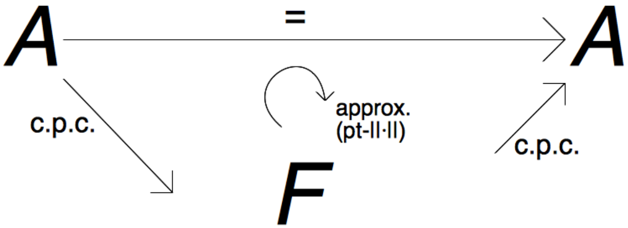

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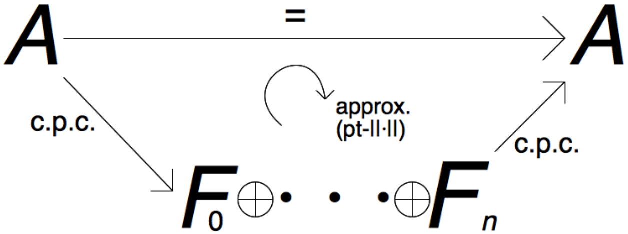

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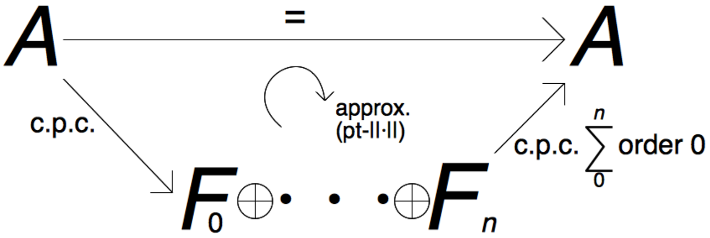

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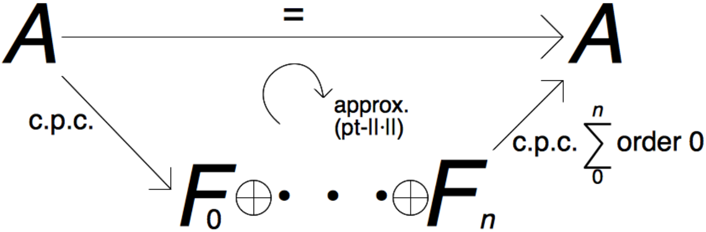

Nuclear dimension is defined by a slight tweaking of the definition of decomposition rank.

While $\mathrm{dr}\,(A) < \infty$ implies A is quasidiagonal, $\dim_{nuc}(\mathcal{O}_n) = 1$ (for $n < \infty$) for example.

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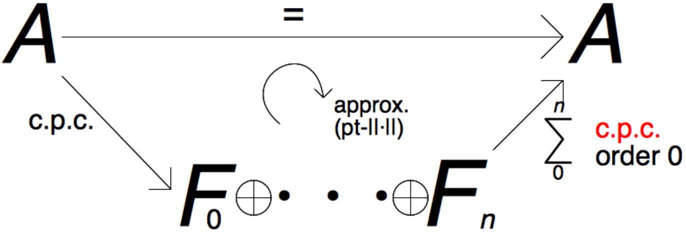

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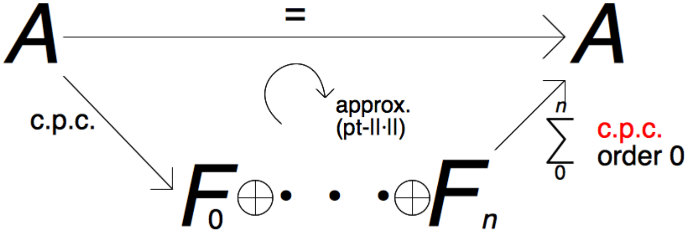

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$\dim_{nuc} < \infty$

strict comparison

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$\Big\Downarrow$ Rørdam ('04)

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$\Downarrow$ $\Uparrow$

with additional hypotheses:
Winter ('12), Matui-Sato (arXiv '11),
Toms-White-Winter

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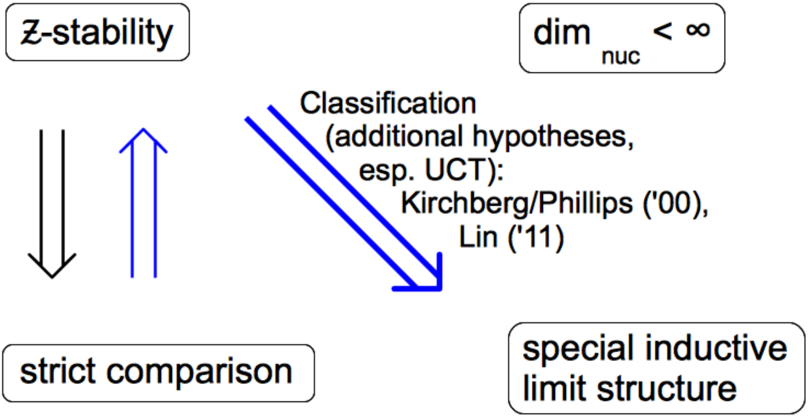

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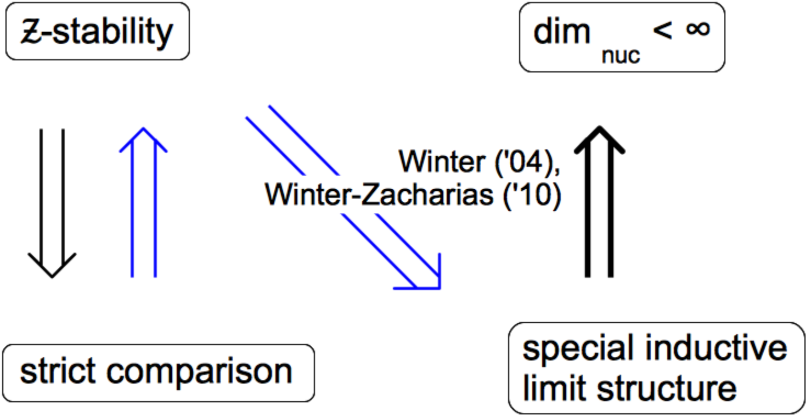

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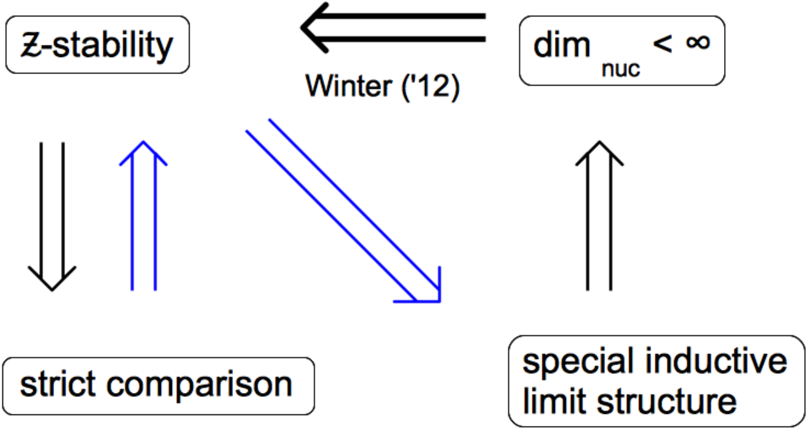

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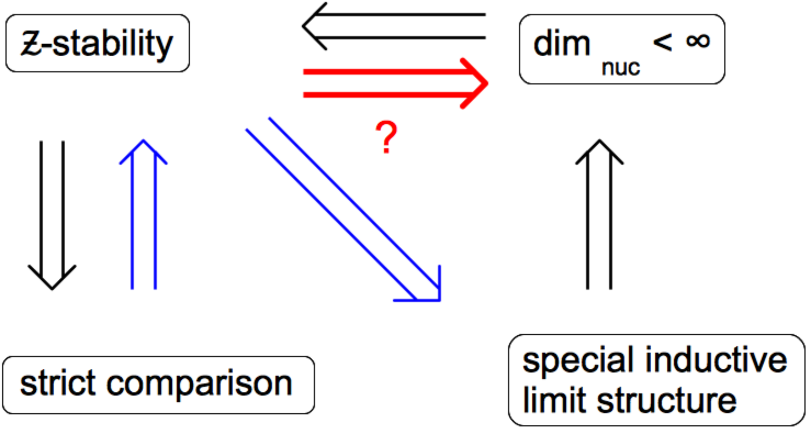

$\mathcal{Z}$-stability and nuclear dimension

A test question for $\mathcal{Z}$-stable $\Rightarrow$ finite nuclear dimension, without classification:

Question

What is the nuclear dimension of $C(X, \mathcal{Z}) = C(X) \otimes \mathcal{Z}$?

On the one hand:
Since
$\dim_{nuc} C(X, M_n) = \dim X$, may expect
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Some key ideas in the proof:

"Tracially" approximate, orthogonal partition of unity in $C(X, M_{n^\infty})_\infty$.

Fill the (tracially small) holes with an embedding of $C_0(Z, \mathcal{O}_2)$,

Embedding exists by quasidiagonality of $C_0((0,1], \mathcal{O}_2)$ (Voiculescu, '91).

Kirchberg-Rørdam ('05): $\dim_{nuc} C_0(Z, \mathcal{O}_2) \leq 3$.

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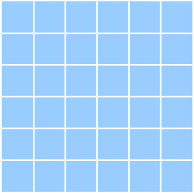

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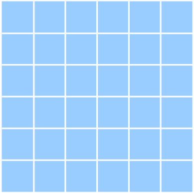
Embedding exists by quasidiagonality of $C_0((0,1], \mathcal{O}_2)$ (Voiculescu, ’91).

Kirchberg-Rørdam (’05): $\dim_{nuc} C_0(Z, \mathcal{O}_2) \leq 3$.

Proof

Some key ideas in the proof:

"Tracially" approximate, orthogonal partition of unity in $C(X, M_{n^\infty})_\infty$.



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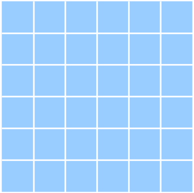

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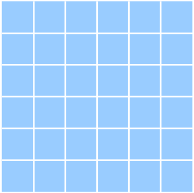

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Is $\dim_{nuc}(A \otimes \mathcal{Z}) < \infty$ for every nuclear C^*-algebra A?
Equivalently, is $\dim_{nuc}(A \otimes \mathcal{Z})$ universally bounded for such A?

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Can we approximate $C(X)$ inside $C(X, M_n)$ in a 2-dimensional way (3 colours)? At least, $<$ dim X dimensions? Or is it necessary to put $C(X)$ into $C(X, M_{n^\infty})$?

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