Nuclearity for partial crossed products by exact discrete groups

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The case of global actions

Anantharaman-Delaroche introduced in mid 80's a notion of amenability for actions of discrete groups: $\alpha\colon G \curvearrowright A$ is *amenable* if the induced action $\alpha''\colon G \curvearrowright A''$ is amenable, or equiv., there is a net of finitely supported positive type functions $\theta_i\colon G \to \mathrm{Z}(A'')$ such that $h_i(e) \leq 1$ and $\theta_i(s) \to 1$ ultraweakly for all $s \in G$.

- If A is nuclear, $\alpha\colon G \curvearrowright A$ is amenable iff $A \rtimes_{\alpha,r} G$ is nuclear (AD).

We say that α has the *weak containment property* if $A \rtimes_\alpha G \cong A \rtimes_{\alpha,r} G$ via the left regular representation.

Theorem (Matsumura)

Let α be an action of an exact discrete group G on a unital commutative C^*-algebra A. If α has the weak containment property, then the reduced crossed product $A \rtimes_{\alpha,r} G$ is nuclear, or equiv., α is amenable.

This also holds for A non-unital (Buss-Echterhoff-Willett).

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Restrictions of global actions

Let G be a discrete group with unit element e and $\beta\colon G \curvearrowright B$ be an action. Let A be an ideal in B. For each $g \in G$, set $A_g := A \cap \beta_g(A)$ and $\alpha_g := \beta_g \restriction_{A_{g^{-1}}}$. Then $\alpha_g\colon A_{g^{-1}} \to A_g$ is an isomorphism for each $g \in G$ and the pair $\alpha = (\{A_g\}_{g\in G}, \{\alpha_g\}_{g\in G})$ has the following properties:

(i) $A_e = A$ and $\alpha_e = \mathrm{id}_A$;

(ii)
$$\begin{aligned}\alpha_g(A_{g^{-1}} \cap A_h) &= \beta_g(A \cap \beta_{g^{-1}}(A) \cap \beta_h(A)) \\ &= \beta_g(A) \cap A \cap \beta_{gh}(A) \subset A_{gh};\end{aligned}$$

(iii) $\alpha_g \circ \alpha_h = \alpha_{gh}$ on $A_{h^{-1}} \cap A_{h^{-1}g^{-1}} = A \cap \beta_{h^{-1}}(A) \cap \beta_{h^{-1}g^{-1}}(A)$;

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Partial actions

A *partial action* of G on a C^*-algebra A is a pair $\alpha = (\{A_g\}_{g\in G}, \{\alpha_g\}_{g\in G})$, where $\{A_g\}_{g\in G}$ is a family of closed two-sided ideals of A and $\alpha_g\colon A_{g^{-1}} \to A_g$ is an isomorphism for each $g \in G$, such that for all $g, h \in G$

(i) $A_e = A$ and α_e is the identity on A;

(ii) $\alpha_g(A_{g^{-1}} \cap A_h) \subseteq A_{gh}$;

(iii) $\alpha_g \circ \alpha_h = \alpha_{gh}$ on $A_{h^{-1}} \cap A_{(gh)^{-1}}$.

Examples

Ex1 Let $I, J \triangleleft A$ and $\alpha\colon I \to J$ a *-isomorphism. Then α induces a partial action $\alpha = (\{A_n\}_{n\in\mathbb{Z}}, \{\alpha_n\}_{n\in\mathbb{Z}})$; here $A_1 = J$ and $A_{-1} = I$; A_n is the domain of α^{-n}.

Ex2 Let X be a LCH space. A partial action $\gamma = (\{\mathrm{C}_0(U_g)\}_{g\in G}, \{\gamma_g\}_{g\in G})$ on $\mathrm{C}_0(X)$ corresponds to a topological partial action $\hat{\gamma} = (\{U_g\}_{g\in G}, \{\hat{\gamma}_g\}_{g\in G})$ on X, that is, $\hat{\gamma}_g\colon U_{g^{-1}} \to U_g$ is a homeomorphism and $\hat{\gamma}_{gh}$ extends $\hat{\gamma}_g \circ \hat{\gamma}_h$. The *-isomorphism γ_g is then

$$f \in \mathrm{C}_0(U_{g^{-1}}) \mapsto f \circ \hat{\gamma}_{g^{-1}} \in \mathrm{C}_0(U_g).$$

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Full partial crossed product

A partial action $\alpha = (\{A_g\}_{g\in G}, \{\alpha_g\}_{g\in G})$ gives rise to a *-algebra as follows. Let $\mathcal{B}_\alpha$ be the set of all finite formal linear combinations

$$\sum_{g\in G} a_g\delta_g \quad (a_g \in A_g).$$

Define multiplication and involution operations on $\mathcal{B}_\alpha$ by setting

$$(a\delta_g)\cdot(b\delta_h) := \alpha_g(\alpha_{g^{-1}}(a)b)\delta_{gh} \qquad (a\delta_g)^* := \alpha_{g^{-1}}(a^*)\delta_{g^{-1}}.$$

The partial crossed product of A by G under α, denoted by $A \rtimes_\alpha G$, is the enveloping C^*-algebra of $\mathcal{B}_\alpha$.

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Weak containment property

The *reduced partial crossed product* $A \rtimes_{\alpha,r} G$ is the C^*-algebra generated by a certain concrete representation $\mathcal{B}_\alpha$. The induced *-homomorphism $\Lambda\colon A \rtimes_\alpha G \to A \rtimes_{\alpha,r} G$ is called the *regular representation* of $A \rtimes_\alpha G$.

- We say that the partial action α has the *weak containment property* if Λ is an isomorphism.

Theorem (Ara–Exel–Katsura)

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Approximation property

Definition (Exel)

A partial action $\alpha = (\{A_g\}_{g\in G}, \{\alpha_g\}_{g\in G})$ has the approximation property if there exists a net of functions $(\xi_i\colon G \to A)_{i\in I}$ with finite support satisfying

(i) $\sup_{i\in I} \|\sum_{g\in G} \xi_i(g)^*\xi_i(g)\| < \infty$;

(ii) $\lim_i \sum_{h\in G} \xi_i(gh)^*\alpha_g(\alpha_{g^{-1}}(a)\xi_i(h)) = a$, for all $g \in G$ and $a \in A_g$.

Theorem (Exel)

If α has the approximation property, then it satisfies the weak containment property. If, in addition, A is nuclear, then $A \rtimes_{\alpha,r} G$ is nuclear, too.

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Equivalence: approximation property and nuclearity

Theorem (Buss–Echterhoff–Willett, Bearden-Crann, Ozawa-Suzuki)

Let $\alpha = (\{A_g\}_{g\in G}, \{\alpha_g\}_{g\in G})$ *be a partial action of* G *on a nuclear* C^**-algebra* A*. If* $A \rtimes_{\alpha,r} G$ *is nuclear, then* α *has Exel's approximation property.*

In particular, for a partial action αon a nuclear C^*-algebra A, we have:

α has the approximation property $\Leftrightarrow$ $A \rtimes_\alpha G$ is nuclear $\Rightarrow$ α satisfies weak containment.

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The main theorem

Theorem (Buss–Ferraro–S.)

Let α be a partial action of an exact discrete group G on a commutative C^*-algebra A. Suppose that α satisfies the weak containment property, that is, $A \rtimes_\alpha G \cong A \rtimes_{\alpha,r} G$ canonically. Then the reduced partial crossed product $A \rtimes_{\alpha,r} G$ is nuclear, or equivalently, α has the approximation property.

Idea: G is exact if and only if $\ell^\infty(G) \rtimes_{\tau,r} G$ is nuclear (Ozawa). Thus G exact $\Rightarrow (\ell^\infty(G) \otimes A'') \rtimes_{\tau\otimes\alpha'',r} G$ is nuclear.
If α satisfies weak containment, we have a canonical inclusion $A \rtimes_\alpha G \hookrightarrow (\ell^\infty(G) \otimes A'') \rtimes_{\tau\otimes\alpha'',r} G$ and we get a ccp map $\varphi''\colon (\ell^\infty(G) \otimes A'') \rtimes_{\tau\otimes\alpha'',r} G \to A'' \rtimes_{\alpha'',r} G$ extending the canonical inclusion. This implies nuclearity of $A'' \rtimes_{\alpha'',r} G$ and hence also of $A \rtimes_{\alpha,r} G$.

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A general characterisation

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Let G be an exact discrete group and let α be a partial action of G on a C^*-algebra A. Then the following are equivalent:

(i) *α has the approximation property;*

(ii) *for every partial action β of G on a C^*-algebra B, we have*

$$(A \otimes_{\max} B) \rtimes_{\alpha\otimes\beta} G = (A \otimes_{\max} B) \rtimes_{\alpha\otimes\beta,r} G.$$

(iii) $(A \otimes_{\max} A^{\mathrm{op}}) \rtimes_{\alpha\otimes\alpha^{\mathrm{op}}} G = (A \otimes_{\max} A^{\mathrm{op}}) \rtimes_{\alpha\otimes\alpha^{\mathrm{op}},r} G.$

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