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COMPLEXITY OF EYE MOVEMENTS IN READING

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During reading, our eyes perform complicated sequences of fixations on words. Stochastic models of eye movement control suggest that this seemingly erratic behavior can be attributed to noise in the oculomotor system and random fluctuations in lexical processing. Here, we present a qualitative analysis of a recently published dynamical model [Engbert *et al.*, 2002] and propose that deterministic nonlinear control accounts for much of the observed complexity of eye movement patterns during reading. Based on a symbolic coding technique we analyze robust statistical features of simulated fixation sequences.

Keywords: SWIFT model; symbolic coding; visual perception.

1. Introduction

Information processing during visual perception is strongly influenced by constraints arising from the human oculomotor system. Only in a small part of the visual field, i.e. within the central 2° of visual angle (the *fovea*), visual acuity permits the analysis of finely structured objects.[1] Consequently, we have to sample information of complicated scenes by many *fixations*, i.e. intervals, during which the eyes perform only microscopic movements [Ditchburn, 1955; Engbert & Kliegl, 2003b], on different parts of an image. These fixations are separated by rapid eye movements called *saccades*. Consequently, our visual system may be looked upon as a combination of a high-acuity detector, the eye, with a high-precision movement system, the oculomotor system [Carpenter, 2000].

The analysis of eye movements in reading provides an interesting case study for the more general problem of information processing of more complex visual scenes. During reading, mean fixation durations on words range roughly from 200 ms to 300 ms, depending on lexical difficulties of words. These fixations are interrupted by saccades with a mean duration between 20 ms and 40 ms [Dodge & Cline, 1901]. Conceptually, the processing of written language appears to occur mostly sequentially from left to right in normal text. This sequential nature of processing does not correspond with the sequence of fixations, which in addition to word-to-next-word movements exhibits multiple fixations on a word (*refixations*), *skippings* of words and *regressions* to previous words (for more details, see Sec. 3.1).

[1]The decrease of visual acuity from the center of the visual field to the periphery is continuous. For a more detailed discussion see [Legge *et al.*, 2001].

Theoretical and computational models have been proposed that account for complex sequences of eye movements on the assumption that a "spotlight of attention" — not the eye — moves sequentially across words and can (re-)program saccades according to local processing demands [Morrison, 1984; Reichle *et al.*, 1998]. Alternatively, the seemingly stochastic nature of eye movements is derived from assumptions of parallel processing of several words in an attentional window [Engbert *et al.*, 2002; Reilly & Radach, 2003]. The overriding goal of these models is to understand the dynamical coupling of lexical processes (e.g. word identification) and oculomotor processes (e.g. visual acuity, programming of saccades).

In this paper, we propose that most of the exceptions from sequential eye movements can be explained by deterministic nonlinear control in a recently published theoretical model of eye movement control during reading called SWIFT[2] [Engbert *et al.*, 2002]. This theoretical model of complex eye trajectories is based on several new principles. The most important concept with respect to the analysis presented here is a separation between saccade timing ("when") and saccade target selection ("where"), which is motivated by recent neurophysiological findings [Wurtz, 1996; Findlay & Walker, 1999; Carpenter, 2000]. After a brief description of the SWIFT model (Sec. 2), we present a qualitative analysis of a deterministic version of this model in Sec. 3. In Sec. 4, we investigate important statistical properties of the simulated fixation sequences, develop new statistical measures of fixation sequences, and discuss implications of our findings for experimental data.

2. The SWIFT Model

In a recently proposed SWIFT model for the control of eye movements in reading ([Engbert *et al.*, 2002], see also [Kliegl & Engbert, 2003]), complex eye movement patterns are generated as an emergent property of the target selection mechanism which operates on a dynamically changing set of variables associated with the sequence of words in a given sentence. We start our analysis with a brief description of SWIFT.

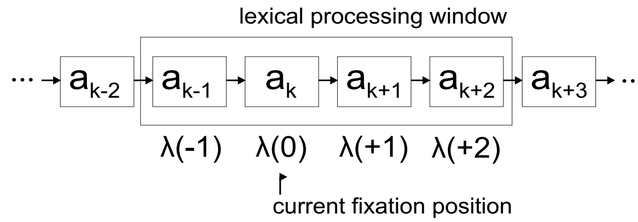


Fig. 1. Eye movements in reading and lexical processing. In our model, four words can be lexically processed from a current fixation position $k(t)$ at time t on word$_k$. Lexical processing rates are related to visual acuity, i.e. $\lambda(0) > \lambda(\pm 1) > \lambda(2)$. The asymmetrical extension of the processing window (one word to the right and two words to the left of the currently fixated word) is motivated by experimental results (see text).

Motivated by recent experimental results [Kennedy, 2000a, 2000b; Inhoff *et al.*, 2000], our model is based on spatially distributed lexical processing, i.e. we assume that the meaning of several words is accessed in parallel. Due to the decrease in efficiency of visual processing from the center to the periphery of the visual field, the currently fixated word is processed faster than words to the left and right of the fixation position $k(t)$ at time t. To include this mechanism in our model, we introduce a lexical processing rate $\lambda > 0$ which is a function of the eccentricity ϵ_n of word$_n$, i.e. the distance of word$_n$ from the fixation position $k(t)$ at word$_k$,

$$\varepsilon_n(t) = n - k(t)\,, \tag{1}$$

where k and n are integers measured in units of words, i.e. word length is not used at this point.[3] We assume that the lexical processing rate $\lambda(\varepsilon)$ reaches its maximum at eccentricity $\varepsilon_k = 0$, i.e. the current fixation position at word$_k$, and decreases to words left and right of word$_k$, i.e. $\lambda(0) > \lambda(\pm 1) > \lambda(\pm 2)$ [Legge *et al.*, 2001]. Furthermore, we assume a vanishing lexical processing rate λ for eccentricities $\varepsilon < -1$ and $\varepsilon > 2$ (Fig. 1). This asymmetry in the spatial extension of the lexical processing window is consistent with research on the so-called *perceptual span* [Rayner, 1998].

To keep track of the state of lexical processing of the set of N_w words in a sentence, we introduce a set of lexical activities $\{a_n(t)\}$ ($n = 1, 2, 3, \ldots, N_w$). The lexical activity $a_n(t)$ of word$_n$ represents a measure of the probability that the word is selected as a saccade target. We use a

[2]**S**accade generation **W**ith **I**nhibition by **F**oveal **T**argets.

[3]It is important to note that longer words have generally lower frequency; therefore, word length is taken into account indirectly.

two-level assumption for lexical processing [Reichle *et al.*, 1998; Engbert & Kliegl, 2001; Engbert *et al.*, 2002]. First, during a preliminary or preprocessing stage, lexical activity is increasing, $da_n(t)/dt > 0$. At this stage, basic properties like word length or initial letters are analyzed.[4] After reaching the maximum L_n of the lexical activity, which we call the lexical difficulty of word$_n$, lexical access continues in a second stage (*lexical completion*) with decreasing lexical activity, $da_n(t)/dt < 0$, until word$_n$ is completely processed when $a_n(t) = 0$.

The lexical difficulty L_n of word$_n$ determines its lexical access time. During reading, there are always discontinous changes of word eccentricities $\varepsilon_n(t)$ due to saccades, which create a complicated time-course of the lexical processing rate $\lambda(\varepsilon_n(t))$. The lexical access time depends on its frequency f_n of occurrence in normal language. As an approximate measure, we use its printed frequency per million words for a corpus of English sentences [Schilling *et al.*, 1998]. In lexically-driven models of eye movement control, lexical processing time is a function of the logarithm of word frequency. As a concurrent process to lexical access from visual input, we are able to predict many words from the context. To include this effect, we use the empirically determined probability p_n to predict word$_n$ from the previous words, i.e. word$_1$, word$_2$, $\ldots$, word$_{n-1}$. It is important to note that the probabilities for predictions p_n represent a considerable amount of our knowledge of language (e.g. syntax, semantics).[5] To capture these competing mechanisms in a first-order approximation, we assume that the lexical difficulty L_n can be written as the product [Reichle *et al.*, 1998; Engbert & Kliegl, 2001; Engbert *et al.*, 2002],

$$L_n = (1 - p_n)(\alpha - \beta \log f_n)\,, \tag{2}$$

where α and β are constant parameters. Using Eq. (2), each sentence of N_w words can be transformed into an input stream of lexical difficulties $L_1, L_2, L_3, \ldots, L_{N_w}$, i.e. a correlated sequence of random variables. Therefore, our model is stochastically forced by correlated noise. For word$_n$, the lexical activity $\{a_n(t)\}$ changes due to lexical processing, where the processing rate λ changes dynamically as a function of the eccentricity $\varepsilon_n(t)$,

$$\frac{da_n(t)}{dt} = \begin{cases} f\lambda(\varepsilon_n(t))\,, & \text{if } t < t_p: \text{ preprocessing} \\ -\lambda(\varepsilon_n(t))\,, & \text{if } t \geq t_p: \text{ lexical completion} \end{cases} \tag{3}$$

with $t_p = \min\{t | a_n(t) = L_n\}$ as the preprocessing time. The preprocessing factor $f > 1$ induces a faster build-up of lexical activity. As a consequence, the eye movement control system will tend to select a word as a saccade target in an early stage of lexical processing.

The set of lexical activities $\{a_n(t)\}$ is a key concept of the SWIFT model, since the current state of lexical processing of a sentence is stored in $\{a_n(t)\}$. Any deficits in lexical processing will be reflected in a non-vanishing lexical activity $a_j(t) > 0$ *after* fixation, which will induce a regression to word$_j$ later in time. With respect to the time-course of saccades, we use the "minimal" assumption of random timing: saccades are generated randomly in time, based on a predefined gamma distribution with mean t_s. To adjust the random time interval to the required processing time for difficult words, we propose a foveal inhibition process ([Engbert *et al.*, 2002], for a mathematical analysis of the inhibition process see [Kliegl & Engbert, 2003]). The time between two subsequent commands to start a saccade program is given by the random time interval t_s and an additive contribution of foveal inhibition $h \cdot a_k(t)$, where h is a constant and $a_k(t)$ is the foveal lexical activity. As a consequence, the next decision to program a saccadic eye movement is generated after the time interval

$$t' = t_s + h \cdot a_k(t)\,. \tag{4}$$

It is important to note that the foveal activity $a_k(t)$ is generally a rapidly decreasing function of time t. Therefore, the contribution of foveal inhibition is finite even for $h \to \infty$. It can be shown by an analytical approximation [Kliegl & Engbert, 2003] that the maximum time interval δt contributed to fixation duration by the inhibition process is

$$\delta t \approx \frac{\alpha}{\lambda(0) + \dfrac{1}{h}} \tag{5}$$

[4] A fast build-up of lexical activity $a_n(t)$ is necessary for early target selection, which turns out to be an essential dynamical principle in word skipping (Fig. 3).

[5] In random word lists, all probabilites vanish, $p_n = 0$. Therefore, the model performs quantitatively differently on random lists of words compared to real sentences. The qualitative principles of the model, however, are the same in both cases of input.

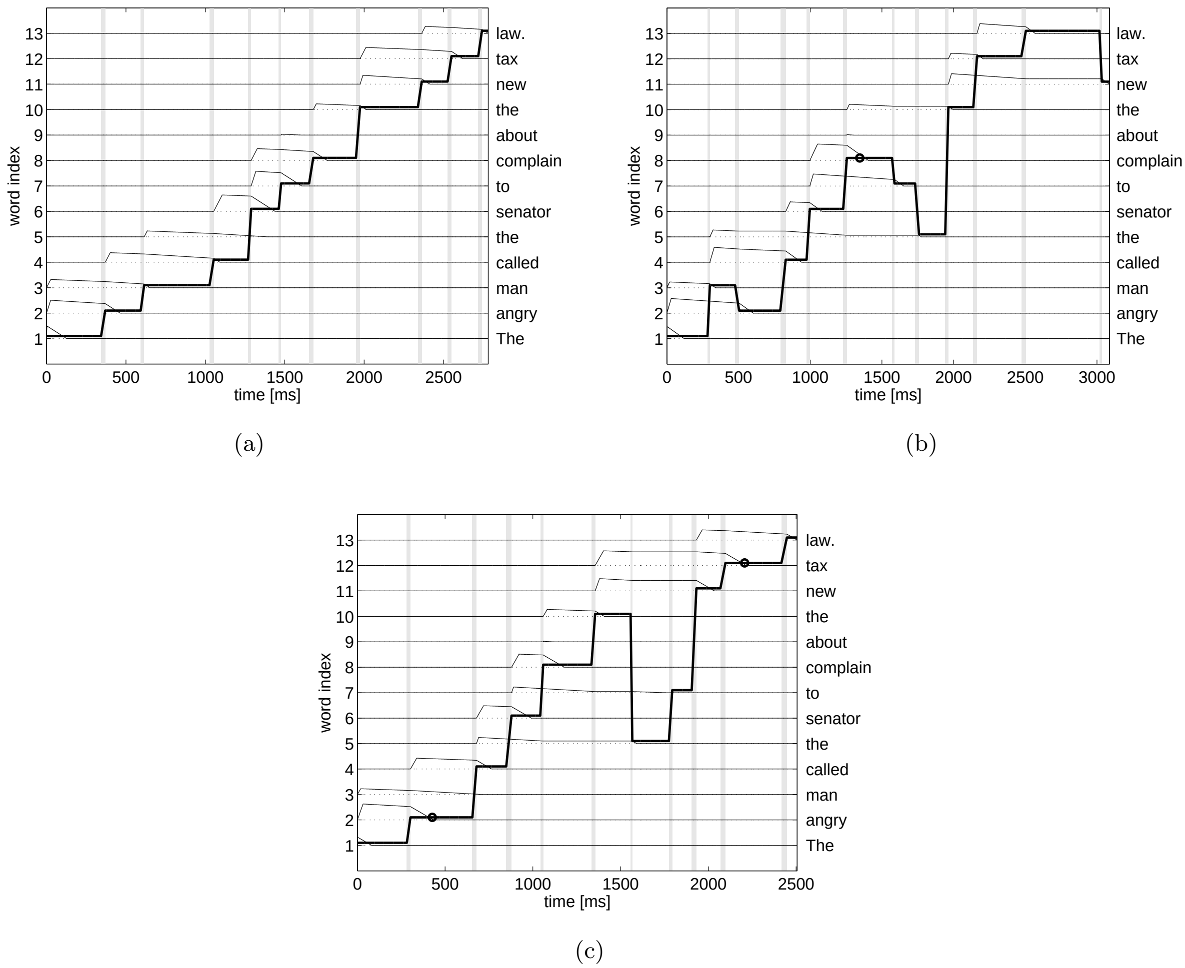


Fig. 2. Examples of numerical simulations of SWIFT. Lexical activities $a_n(t)$ of the words to the right of the illustrations are plotted on the vertical axis (thin lines). (a) A simulation with a sequential eye movement trajectory (bold line). Note that words 5 and 9 are skipped in this example. (b) A second run on the same sentence shows four regressions, i.e. movements of the eyes from right to left, to words 2, 5, 7 and 11. A refixation on word 8 is indicated by the circle. Word 9 is skipped. (c) In the third example, we observe refixations on words 2 and 12; words 3 and 9 are skipped and regressions hit words 5 and 7.

In the case that a new command to program a saccade arises before the current program terminates, the saccade is canceled. Since each cancelation resets the saccade program and induces an additional fixation time, however, the cancelation process is limited to two cancelations per saccade program. The programming of a saccade is divided into two different stages, a *labile* stage of duration τ_l, during which the program can be canceled, and a *non-labile* stage of duration τ_n, which, when terminated, mandatorily induces the execution of a saccade of duration τ_{ex}.

Additionally, we assume that stochasticity plays a central role for saccade target selection: saccade targets are selected randomly, but with probability weights computed from lexical activities $\{a_n(t)\}$. Saccade target selection is performed after the latency $\tau_{\rm tar}$ during the labile stage.[6] The conditional probability $\pi(n,t|k)$ for word$_n$ to be selected as a saccade target at time t, if the current fixation position is $k(t)$, is given by its relative lexical activity, i.e.

$$\pi(n,t|k) = \begin{cases} \dfrac{a_n(t)}{\displaystyle\sum_{m=1}^{k+2} a_m(t)}\,, & \text{if } n \le k+2 \\ 0\,, & \text{if } n > k+2\,, \end{cases} \qquad (6)$$

[6]This random variable is implemented as a random fraction of the labile program τ_l, since we assume that target selection happens during the labile stage.

if $\sum_{m=1}^{k+2} a_m(t) > 0$. Whenever the denominator in Eq. (6) vanishes, we select as a saccade target the next closest word to the right of the current processing window, which is not (or not completely) lexically processed.[7]

Due to the dynamic target selection mechanism, Eq. (6), the SWIFT model inherently generates complex eye movement trajectories. Typical simulations are shown in Fig. 2. The sentence in Fig. 2 is taken from a corpus of 48 sentences [Schilling *et al.*, 1998], for which word frequencies f_n and prediction probabilities p_n were provided by Reichle *et al.* [1998]. Model parameters are $\alpha = 148.5$ ms, $\beta = 5.71$ ms, $f = 62.5$, $\lambda(0) = 0.798$, $\lambda(-1) = \lambda(1) = 0.077$, $\lambda(2) = 0.048$, $t_s = 187.1$ ms, $\tau_l = 128.6$ ms, $\tau_n = 41.6$ ms and $\tau_{\text{tar}} = 112.1$ ms. Saccade intervals t_s, lexical processing difficulties L_n, and saccade program latencies τ are gamma-distributed random variables with a proportion of $\rho = 0.239$ between standard deviations and mean values. Saccade execution time is chosen as $\tau_{ex} = 25$ ms, where the standard deviation is fixed at 1/3 of the mean. For details of the parameter estimation procedure see [Engbert *et al.*, 2002].

One of the key problems in eye movement research is the question what factors are affected when and where we make a regressive saccade during reading [Liversedge & Findlay, 2000; Starr & Rayner, 2001]. Figure 2 illustrates that SWIFT predicts the occurrence of regressions due to lexical processing deficits. In the model simulations, several words are not completely processed in first-pass, i.e. lexical activities do not vanish after fixation or skipping. Due to the target selection algorithm, these nonvanishing lexical activities will induce regressions later in time. This type of behavior is observed in the case of word$_2$, word$_5$ and word$_{10}$ in Fig. 2(b) and word$_5$ and word$_7$ in Fig. 2(c). In the next section, we develop a technique for the analysis of fixation sequences based on symbolic codings.

3. Qualitative Analysis by Symbolic Dynamics

The variability in fixation sequences poses a difficult problem for a quantitative analysis of eye movement data in reading. Because there is no straightforward way to compare different eye movement paths (e.g. Fig. 2), statistical analyses are traditionally restricted to experimental trials, where participants moved their eyes from left to right [Rayner, 1998]. Nevertheless, statistical analyses of fixation durations contributed significantly to our understanding of eye movement control in reading. At least two shortcomings of data preselection related to sequential processing models, however, are obvious. First, this procedure causes a loss of roughly 2/3 of all experimental reading data.[8] Second, data preselection makes is difficult to generalize models to more general eye movement patterns. Consequently, advances in modeling of eye movement control were limited to sequential aspects without a theoretical link to the problem of regressions [Reichle *et al.*, 1998]. Therefore, we can expect interesting new insights from a complementary analysis, in which we address the statistical analysis of fixation sequences and discard information on fixation durations [Hogaboam, 1983].

In this paper, we restrict our analysis to fixation sequences of numerical simulations of the SWIFT model [Engbert *et al.*, 2002]. We present a qualitative dynamical analysis of our model (described in Sec. 2), based on a symbolic description of fixation sequences, and show that model simulations are an ideal starting point to derive new measures of eye movement patterns for the analysis of experimental data.

3.1. *Symbolic coding of fixation sequences*

Symbolic dynamics was developed to study dynamical properties of nonlinear dynamical systems [Hao, 1991; Guckenheimer & Holmes, 1993; Badii & Politi, 1997]. For practical purposes, symbolic representations turned out to be a powerful tool to extract robust properties of underlying dynamical processes, in particular, for complex biological systems, which often generate nonstationary and/or noisy time series (e.g. [Engbert *et al.*, 1997; beim Graben *et al.*, 2000], and references therein).

Before we introduce a symbolic coding for fixation sequences, we give a brief description of some terminology for a characterization of eye movements in reading. A *single fixation* is the most frequent

[7] The simulation is terminated, if all words are completely processed, i.e. $a_n(t) = 0$ for all $n = 1, 2, 3, \ldots, N_w$.

[8] About 10 to 15% of all saccades are regressions [Rayner, 1998]. Assuming a mean number of 10 words per sentence, the probability that no regression occurs in reading of a sentence is smaller than $(1 - 0.1)^{10} = 0.35$.

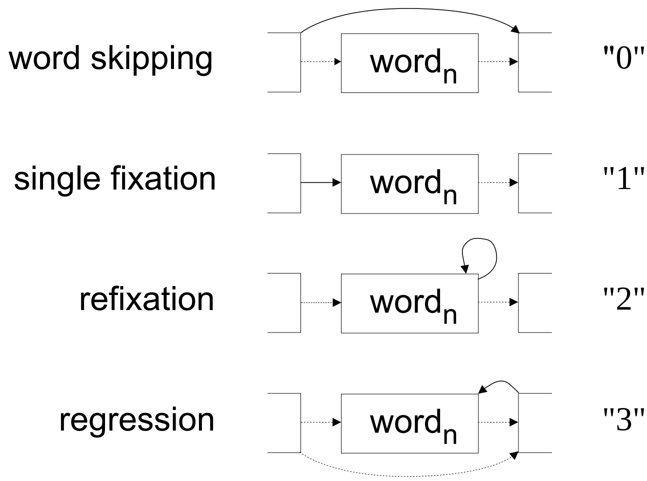


Fig. 3. Symbolic coding of fixation sequences. Depending on the occurrence of four different events in a fixation sequence, we introduce a symbolic coding for each word$_n$ of a sentence. When a word is not fixated, this *word skipping* is coded by the symbol "0" (right column). The most frequent case, a *single fixation*, is mapped to the symbol "1". A second and/or more fixations on the same word are called *refixations*, which we characterize by the symbol "2". Finally, a *regression*, defined as a fixation after a movement from right to left, is coded by symbol "3". In addition to the case in the illustration, a regression can be longer than one word.

pattern observed in eye movements during reading. Aside from strictly sequential (word-to-next-word) eye movements from left to right there are *word skippings*, *refixations*, and *regressions*. Many high-frequency words (e.g. articles) can be lexically processed completely by parafoveal preview, i.e. without fixating the word, so that these words are skipped by eye movements (Fig. 3). Many difficult (or long) words are lexically processed using multiple fixations; the second fixation and all subsequent fixations are called *refixations*.[9] The most dramatic disturbances in fixation sequences are *regressions* (Fig. 3). It is important to note the difference between refixations and regressions: Refixations are defined as multiple fixations of the *same* word, which are not interrupted by fixations on other words, while regressions are fixations on words, which were already fixated or skipped previously.

For a classification of different fixation sequences, we introduce a symbolic coding (Fig. 3, right column). Let us consider a sentence consisting of N_w words. We construct a symbolic coding of the sequence of fixations by mapping one of four symbols ("0", "1", "2" or "3") to each word$_n$ to characterize the sequence.[10] Using this coding scheme, each fixation sequence, consisting of an arbitrary number of fixations, gives a symbol sequence of length N_w. To illustrate some examples during reading of the same sentence, the eye trajectories discussed in Fig. 2 generate the following symbol sequences:

$$\begin{array}{lll}
\text{(a)} & (1)(2)(3)(4)(6)(7)(8)(10)(11)(12)(13) & \mapsto 1111011101111\,, \\
\text{(b)} & (1)(3)(2)(4)(6)(8)(8)(7)(5)(10)(12)(13)(11) & \mapsto 1311313201311\,, \\
\text{(c)} & (1)(2)(2)(4)(6)(8)(10)(5)(7)(11)(12)(12)(13) & \mapsto 1201313101121\,.
\end{array}$$

Each of these symbol sequences represents a number written in a base-4 notation. As a consequence, we characterize each fixation sequence by a single number. First, we use this representation for qualitative analysis of the deterministic SWIFT model. In a second step, we investigate statistical aspects of fixation sequences in the fully stochastic model.

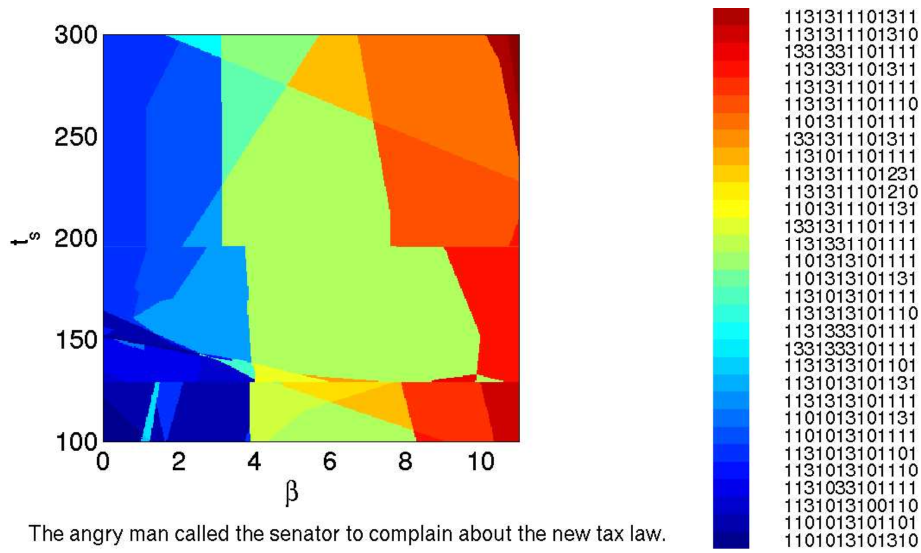
3.2. *Isosequence analysis*

Stochasticity plays an important role on several levels of mathematical models of eye movement control in reading [Reichle *et al.*, 1998; Reilly & O'Regan, 1998; Engbert & Kliegl, 2001; Engbert *et al.*, 2002; Reilly & Radach, 2003]. In complex systems, stochastic fluctuations typically interact with nonlinear control processes and create interesting new phenomena, such as stochastic resonance which has been proposed recently for reading dynamics in a theoretical analysis [Engbert & Kliegl, 2003a].

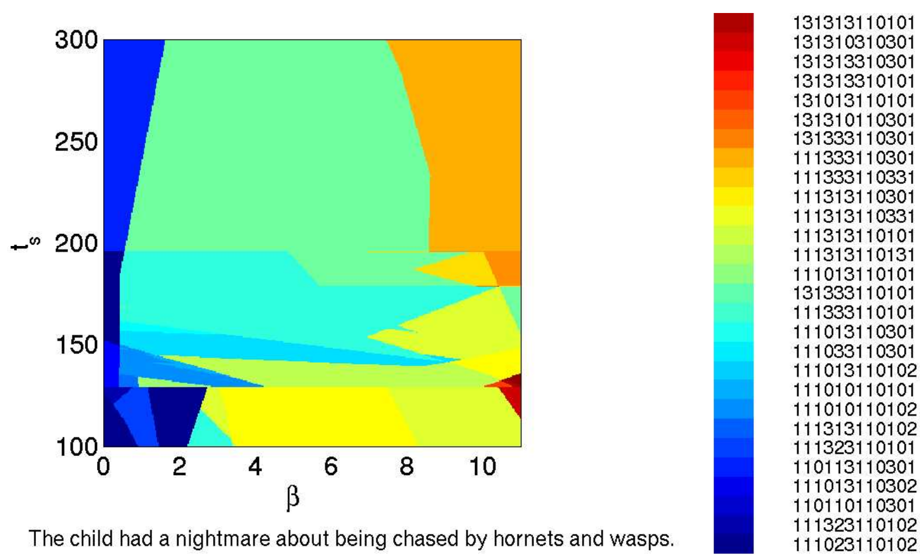
In the case of symbolic codings of fixation sequences, we have found (see Sec. 4) that stochastic influences will lead to a strong increase in

[9] In general, skipping of words as well as refixations are due to cognitive *and* visuomotor factors (e.g. [Brybaert & Vitu, 1998]). However, it is beyond the scope of the current study to discuss the contribution of oculomotor processes to these types of eye movements.

[10] The resulting coding of a fixation sequence into a symbol sequence of length N_w is, in this example, not invertible.



(a)



(b)

Fig. 4. Isosequence plots of the deterministic SWIFT model for two different sentences of the corpus by Schilling *et al.* [1998] [(a) sentence 6, (b) sentence 37]. The slope parameter β of the relation between word frequency and lexical difficulty, Eq. (2), is varied on the abscissa. In the limiting case of $\beta = 0$, lexical difficulty is constant. The mean saccade interval t_s is varied on the ordinate between 100 ms and 300 ms. The color coding indicates the generated symbol sequence (panel on the right). The analysis shows that a rich variety of fixation sequences is generated by the SWIFT model already in the deterministic version because of its nonlinearity. (The color coding is arbitrary.)

the number of realized fixation sequences. Due to target selection based on relative lexical activities, however, the SWIFT model generates complex eye movement sequences without stochasticity. Therefore, we start our investigation with a deterministic version of the SWIFT model. In the deterministic version of our model, all fluctuating parameters (lexical difficulties, saccade latencies, saccade intervals) are kept constant at their mean values, i.e. the proportion between standard deviation and mean is fixed at $\rho = 0$. A different source of stochasticity is the target selection process, Eq. (6). To obtain a deterministic target selection we introduce a "winner-takes-all" mechanism, where the word with highest lexical activity is chosen as the next saccade target with certainty. Therefore, we modify Eq. (6) to

$$\pi(n,t) = \begin{cases} 1\,, \text{if } n = l \quad \text{with} \quad a_l(t) = \max_j\{a_j(t)\} \\ 0\,, \text{otherwise}\,. \end{cases} \tag{7}$$

In the deterministic version, SWIFT generates a unique fixation sequence for each sentence.

To analyze the deterministic "skeleton" of the SWIFT model, we investigate regions of a subspace of control parameters with respect to the symbol sequences generated. In our exploratory analysis, we aim at a variation of model parameters which is related to interindividual differences, since different fixation sequences are experimentally observed in reading trials of the same sentence by different participants. Two such parameters of the model are the mean saccade interval t_s and the slope parameter β of the relation between word frequency and lexical difficulty, Eq. (2). Differences in these two parameters mainly reflect interindividual variability, since the mean saccade interval t_s is strongly related to the reading speed a participant has chosen for a given text and the slope parameter β is related to the vocabulary of the participant, since β controls the decrease of word difficulty with increasing word frequency.

In a resulting 2D plot with variation of β on the abscissa and variation of t_s on the ordinate, we use a color coding for the visualization of realized sequences (Fig. 4). Regions of the plane with the same color correspond to the same symbol sequence, i.e. the same fixation sequence. We call this representation an *isosequence plot*. Figures 4(a) and 4(b) show the isosequence plots for two sentences of the corpus by Schilling *et al.* [1998] [(a) sentence 6, (b) sentence 37]. First, the analysis shows that a rich variety of fixation sequences is generated by the deterministic SWIFT model as two of its important parameters are varied. Therefore, nonlinear determinism in SWIFT adds a completely new source of variability to previous stochastic models.

Second, the number of possible symbols in a sequence depends strongly on the position of the word in the sentence, i.e. the set of possible symbol sequences is constrained by the input sentence. As an example, in sentence 6 [Fig. 4(a)] word$_8$ ("complain") is always processed in a single fixation, while word$_9$ ("about") is always skipped. This result can be explained by nonlinear effects in lexical processing of neighboring words. Since word$_8$ "complain" is a low-frequency word ($f_8 = 11$), the word is fixated with probability one in the deterministic model. Due to the processing window (Fig. 1), however, a fixation on word$_8$ mandatorily induces parafoveal lexical processing of word$_9$. Because the frequency of word$_9$ ("about") is relatively high ($f_9 = 1814$, i.e. two orders of magnitude higher than f_8), word$_9$ is completely processed by parafoveal information.[11] As a consequence, word$_9$ is always skipped in the deterministic model. Because spatially distributed lexical processing is a key feature of SWIFT, we can expect ubiquitous nonlinear dependences in the generated symbol sequences, which we address in the next section.

4. Statistics of Fixation Sequences

In the case of the stochastic SWIFT model [Engbert *et al.*, 2002], the symbolic coding leads to an even larger number of symbol sequences than observed in the deterministic case for the subspace (t_s, β). Therefore, a statistical analysis must be carried out to explore possible structures of fixation sequences. The stochastic SWIFT model potentially generates fixation sequences which are very similar to experimentally observed sequences. Consequently, we restrict our analysis here to numerically simulated data.

Using a rank-frequency analysis we obtain a power-law for the distribution of fixation sequences. In Fig. 5, we performed 10,000 runs (each with a different set of random numbers) of the SWIFT model using sentence 6 [Fig. 5(a)] and sentence 37 [Fig. 5(b)] of the corpus by Schilling *et al.* [1998]. In both cases, we obtain a power-law relation between frequency f and rank number r, i.e. $f(r) \sim r^{-\gamma}$ with $\gamma = 0.82$ for sentence 6 and $\gamma = 0.88$ for sentence 37.

To estimate the effect of spatial correlations between symbols at different position in a sequence, we perform an analysis using surrogate sequences. To this aim, we compute the probability $p_s(w)$ of occurrence of symbol s $(= 0, 1, 2, 3)$ at each position w in the sequences (Table 1). Assuming that the occurrences of symbols are uncorrelated between positions, we construct 10,000 surrogate symbol sequences with the same proba-

[11] The empirically determined predictabilities make the difference of lexical difficulties L_n between these two words even stronger, $p_8 = 0.35$ and $p_9 = 0.95$ [Schilling *et al.*, 1998].

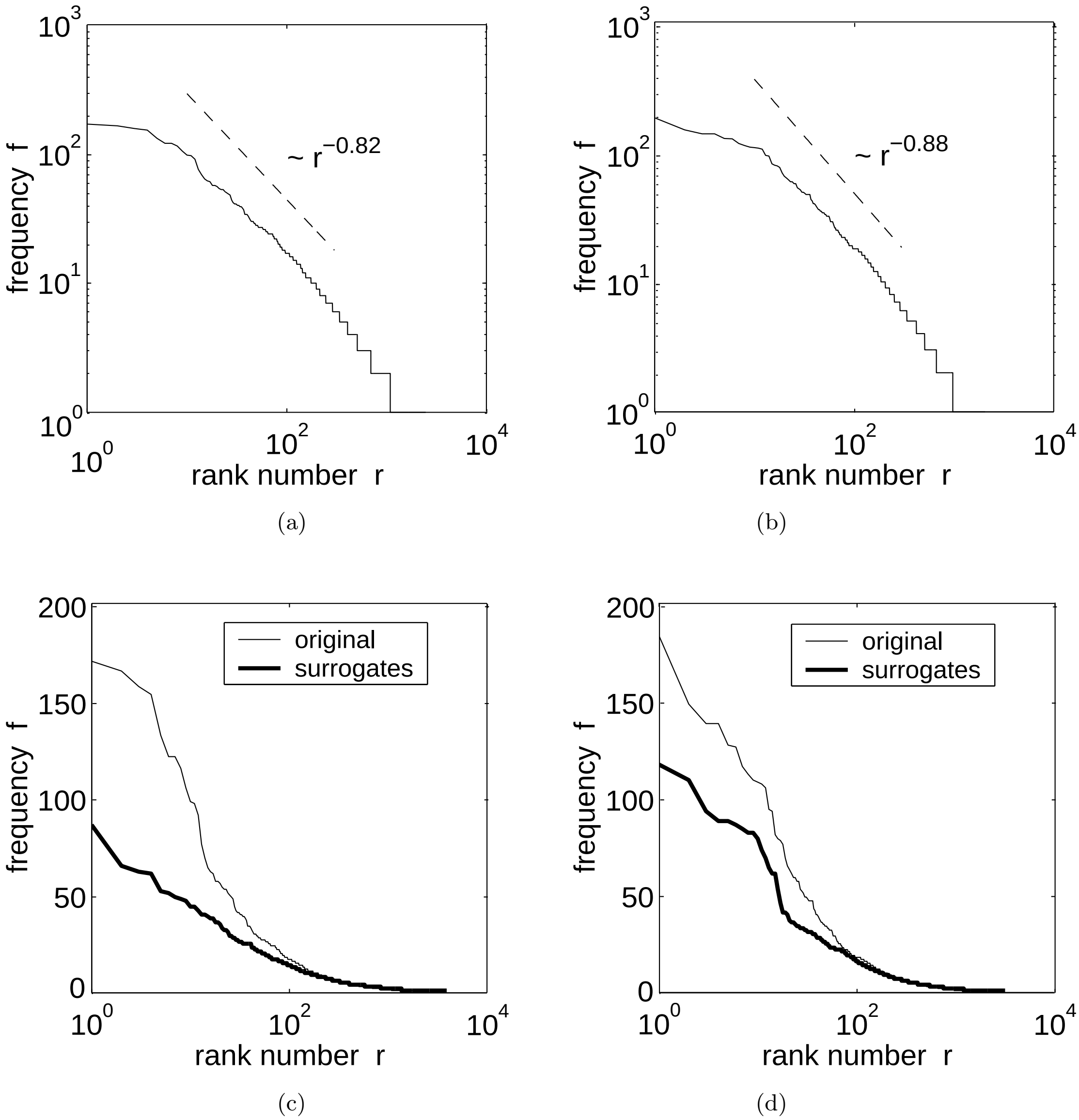


Fig. 5. A power-law for the rank-frequency relation of symbol sequences. (a) The plot shows the rank-ordered frequency of symbol sequences generated by 10,000 runs of the stochastic SWIFT model on sentence 6 [Fig. 4(a)]. The relation between frequency f and rank number r can be approximated by a power-law, i.e. $f(r) \sim r^{-\gamma}$, where $\gamma = 0.82$. (b) In the same analysis as in (a) for sentence 37 [Fig. 4(b)] we obtain $\gamma = 0.88$. (c) A comparison of the distributions between original and surrogate sequences (see text) for sentence 6 indicates that spatial correlations across the sentence lead to an even higher contribution of the most frequent symbol sequences. (d) Same analysis as in (c) for sentence 37.

bility distribution $p_s(w)$.[12] The comparison of the distributions obtained by SWIFT simulations and surrogate technique [Figs. 5(c) and 5(d)] indicates the presence of spatial correlations between symbols across the sentence, which represent correlations between words. These correlation lead to an increase in the number of the most frequent symbol sequences. Thus, an analysis by the technique of symbolic coding might help to investigate and interpret spatial correlations in fixation sequences.

5. Discussion

We investigated the complexity of fixation sequences in reading based on a recently published dynamical model of eye movement control called

[12]This can be implemented on a computer by the linear selection algorithm (e.g. [Engbert & Drepper, 1994]).

Table 1. Probability $p_s(w)$ of symbol s at position w (computed from 10,000 runs of SWIFT performed on sentence 6).

Position	Probability $p_s(w)$ [%]			
w	$s = 0$	$s = 1$	$s = 2$	$s = 3$
1	0	100	0	0
2	0	55	7	38
3	9	48	0	42
4	0	78	8	14
5	19	32	0	49
6	0	71	11	18
7	3	56	2	39
8	0	93	6	1
9	99	1	0	0
10	0	98	1	0
11	12	54	5	30
12	26	67	3	4
13	52	46	2	0

SWIFT [Engbert *et al.*, 2002]. Due to spatially distributed lexical processing and stochastic target selection, SWIFT inherently generates complicated sequences, including word skippings, refixations and regressions.

First, we presented a qualitative analysis of the nonlinear deterministic control principles of a deterministic version of SWIFT. For this analysis, we introduced a symbolic coding of fixation sequences. The results indicate that, even a deterministic version of SWIFT generates rich dynamical behavior without stochasticity due to nonlinear interactions between word streams and inherent dynamics. This nonlinearity arises dynamically from spatially distributed processing. Our results suggest that the experimentally observed complexity of fixation sequences is generated by nonlinear processes in addition to the stochastic influences present at several levels.

Second, for fixation sequences in the stochastic SWIFT model, we carried out a statistical rank-frequency analysis and obtained a power-law relation between rank number r and frequency f. This novel result represents a robust and testable prediction on experimental data. Finally, we investigated correlations in fixation sequences by an analysis using surrogate symbol sequences. Using the null hypothesis that the probabilities of occurrences of the different symbols are uncorrelated between positions, we constructed a large number of surrogates sequences and compared the resulting frequency distribution with the distribution obtained from SWIFT simulations. Our results show that spatial correlations across the sentence give rise to the observed power-law statistics.

The technique of fixation sequence analysis may prove to be useful for the analysis of experimental data and future comparisons between different theoretical models and experimental approaches to the study of eye movement control during reading, visual search, and scene perception.

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