

THIS SAMPLE EXAM IS BASED ON THE FINAL EXAM FROM 2004.

Some questions, on topics not related to the material we've discussed in this course, have been deleted and replaced with more relevant questions; all questions on this exam should therefore seem reasonable to you. Enjoy!

- Duration: 3 hours
- Only calculators in the TI 30/34 series are permitted. Graphing and/or programmable calculators are **not** permitted. Calculators with differentiation or integration capabilities are **not** permitted. Notes or books are **not** permitted.
- Problems 1-9 [1 or 2 points each] are multiple choice. Circle the correct answer. Numerical answers are rounded to the given decimal.
- Problem 10 [3 points] is True-False. Circle the letters of true statements; cross out the letters of false statements.
- Problems 11-22 [2 points each] are short answer type. Write your solutions in the space provided.
- Problems 23 and 24 [8 points total] require detailed and clearly presented solutions. Show all work.
- Work the problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper. Work on multiple choice problems will be examined only in case of suspected fraud.

1.[2 points] For what value of x in the interval $[-\pi/2, \pi/2]$ does the graph of

$$y = \sqrt{3}x + 2 \cos(x)$$

have a horizontal tangent ?

- A. $-\pi/3$ B. $-\pi/6$ C. 0 D. $\pi/6$ E. $\pi/3$ F. never

2.[2 points] Find the slope of the tangent line to the curve $4 \cos(x) \sin(y) = 1$ at $(\pi/3, \pi/6)$.

- A. $1/2$ B. $-1/2$ C. 1 D. -1 E. 0 F. undefined

3.[2 points] Suppose $h(x) = f(g(x))$, $g(2) = 3$, $g'(2) = 4$, $f(2) = 3$, $f'(2) = 6$, $f'(3) = 5$. Find $h'(2)$.

- A. 20 B. 24 C. 12 D. 30 E. 18 F. 15

4.[1 point] If $f(x)$ is a continuous function such that

$$\int_1^7 f(x) dx = 9 \quad \text{and} \quad \int_4^7 f(x) dx = 15,$$

what is $\int_1^4 f(x) dx$?

- A. -6 B. 0 C. 6 D. 9 E. 15 F. 24

5.[2 points] Newton's Method is being used to find a root of $f(x) = x^4 + 6x - 4$ with $x_1 = 1$. What is the value of x_2 ?

- A. 1.30 B. 0.65 C. 0.95 D. 0 E. 1.05 F. 0.70

6.[2 points] An ice cube (shaped like a perfect cube with sides of length 5 cm) is dropped in the Rideau Canal. As the cube melts, its sides decrease in length at a rate of 0.2 cm/minute. How fast is the volume decreasing when the cube has volume 8 cm^3 ?

- A. $8 \text{ cm}^3/\text{minute}$ B. $0.008 \text{ cm}^3/\text{minute}$ C. $0.8 \text{ cm}^3/\text{minute}$
D. $2.4 \text{ cm}^3/\text{minute}$ E. $5.4 \text{ cm}^3/\text{min}$ F. $38.4 \text{ cm}^3/\text{minute}$

7.[2 points] Evaluate $\int_0^2 |2 - e^x| dx$.

- A. $e^2 - 5$ B. $e^2 + 5$ C. $e^2 + 4 \ln 2 - 7$ D. $5 - e^2$ E. $4 \ln 2 - e^2$ F. $|2 - e^x|$

8.[2 points] State the definition of “f is differentiable at $x = x_0$.” Give an example of a function which is not differentiable at $x = 0$.

9.[2 points] Suppose $F(x) = \int_2^x \cos(\ln t) dt$. What is $F'(x)$? What is $F''(x)$? Finally: what is $F''(3)$?

- A. 0.4548 B. 0.9998 C. -0.0192 D. -0.2969 E. -0.0064 F. -0.8906

10.[3 points] Circle the letters of statements that are true. Cross out the letters of statements that are false.

A. The First Derivative Test says that if f is a function and $f'(a) = 0$, then f has a local extremum at a .

B. The Fundamental Theorem of Calculus says that if you have an antiderivative F of f on $[a, b]$, you can use F to compute $\int_a^b f(x) dx$.

C. An inflection point for a function f is a point where the second derivative is 0.

D. If $f'(x) = g'(x)$ for all x on interval I , then $f(x) = g(x)$ on I .

E. f has a local minimum at a if and only if $f''(a) > 0$.

F. The function $f(x) = \arctan x$ is defined for all real numbers x .

11.[2 points] Find $\int \arctan x \, dx$.

12.[2 points] Find $\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos(x) - 1}$.

13.[2 points] Use the Midpoint Rule with $n = 4$ to approximate the integral $\int_{-1}^0 \tan(x^2) dx$. Write your answer to 4 decimal places.

14.[2 points] Find $\int x^2 \sin(x^3) dx$.

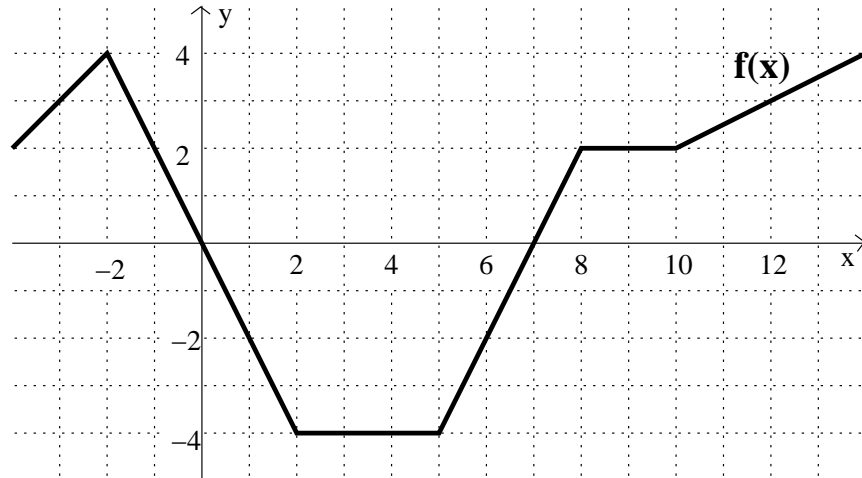
15.[2 points] On which interval(s) is the function $f(x) = \frac{\ln x}{x}$ concave up ?

16.[2 points] When a person coughs, their trachea contracts. A mathematical model for the velocity v (in cm^3/s) of the airstream in the trachea, as a function of the tracheal radius r (given in cm), is

$$v(r) = 7(3 - r)r^2 \quad \text{where } 1.5 \leq r \leq 3.$$

Find the value of r in this interval for which $v(r)$ is at a maximum, and give also this maximum value.

17.[2 points] The graph of a function $y = f(x)$ is given below. Use the graph to calculate $\int_{-2}^5 f(x) dx$ and $\int_{-2}^{10} f(x) dx$.



18.[2 points] Find $\int \frac{1}{x^2+8x+19}$. Hint: complete the square.

19.[2 points] Find $\int \frac{\sqrt{3 + 2 \ln x}}{x} dx$.

20.[2 points] Let $y = x^{\arcsin(x)}$. Find dy/dx .

21.[2 points] Find $\int x^2 e^x dx$.

22.[2 points] Find $\int \frac{2x - 2}{x^2 - 2x - 3} dx$.

23.[3 points] An open top cylindrical can is to be made from a sheet of tin and must hold exactly 1 L of liquid. Determine the dimensions, *ie* the height and the radius of the base, that will require the least amount of tin. Justify why your answer is a minimum.

24.[5 points] Consider the function $f(x) = xe^{-x}$. Note that $\lim_{x \rightarrow \infty} xe^{-x} = 0$. In this question, you are asked to find the first and second derivatives of the function f and to find all important features of its graph. Please work out the details on the bottom of this page and on the next page, and transcribe your final answers neatly to the spaces provided below. Note that a correct answer to some questions may be “none”. Sketch the graph of the function on the next page. Label the important features on your graph.

1. x -intercepts:
2. y -intercept:
3. vertical asymptotes:
4. horizontal asymptotes:
5. $\lim_{x \rightarrow -\infty}$:
6. $f'(x) =$
7. $f''(x) =$
8. interval(s) where the function is increasing:
9. interval(s) where the function is decreasing:
10. local extrema (with (x, y) coordinates and type - max or min):
11. interval(s) where the function is concave up:
12. interval(s) where the function is concave down:
13. inflection point(s)(with (x, y) coordinates):

Extra workspace for Problem 24.