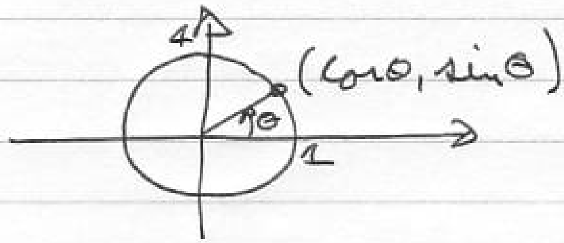
MAT 1726

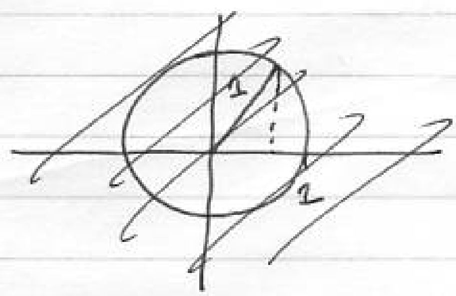
TRIGO :



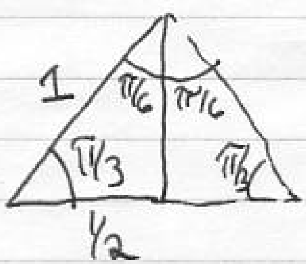
en particulier :

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\cos\theta$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin\theta$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0

$\theta = \pi/3 \ (60^\circ)$



Rappel : • la somme des angles dans un triangle de $\mathbb{R}^2$ est $180^\circ = \pi$



$\Rightarrow \underline{\cos \pi/3} = 1/2$

• $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \underline{\sin \pi/3} = \sqrt{1 - 1/4} = \frac{\sqrt{3}}{2}$

$$\sin \pi/6 = \cos \pi/3 = 1/2$$

$$\cos \pi/6 = \sin \pi/3 = \sqrt{3}/2$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$	-1
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

θ	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$\cos\theta$	$-\sqrt{3}/2$	$-\sqrt{2}/2$	$-1/2$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\sin\theta$	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$	-1	$-\sqrt{3}/2$	$-\sqrt{2}/2$	$-1/2$	0

<u>Rappels</u>:

$$\begin{cases} \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \\ \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \end{cases}$$

- $\sin(2x) = \sin(x+x) = 2\sin x \cos x$ *
- $\cos(2x) = \cos(x+x) = \cos^2 x - \sin^2 x$ *

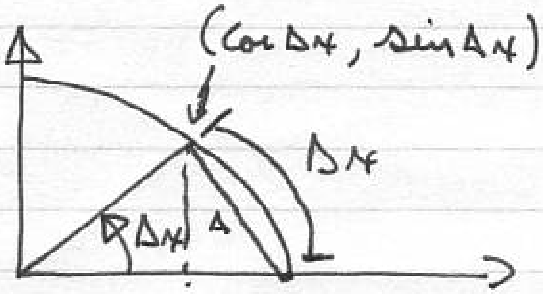
$$= 2\cos^2 x - 1 \quad *$$

$$= 1 - 2\sin^2 x \quad *$$

Lemme 4.27 (p.226)

$$\lim_{\Delta x \to 0} \frac{\cos(\Delta x) - 1}{\Delta x} = 0$$

DEM:



Pythagore $\rightarrow$

$$A = \sqrt{\sin^2 \Delta x + (1 - \cos \Delta x)^2}$$

en particulier

$$0 < A < \Delta x$$

$$0 < A^2 < \Delta x^2$$

$$A^2 = \sin^2 \Delta x + 1 - 2\cos \Delta x + \cos^2 \Delta x$$

$$= 2(1 - \cos \Delta x)$$

$$\Rightarrow \quad 0 < 2(1 - \cos \Delta x) < \Delta x^2$$

$$\Leftrightarrow \quad \boxed{0 > \frac{\cos \Delta x - 1}{\Delta x} > -\frac{\Delta x}{2}}$$

Mais $-\frac{\Delta x}{2} \to 0$ lorsque $\Delta x \to 0$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} = 0$$ ∎

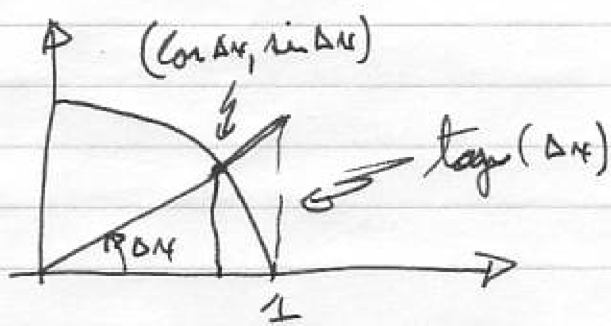
<u>Lemme 4.28</u>: $\lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} = 1$

<u>Remarque</u>: le résultat précédent montre que $\cos x$ est diff. en 0

$\Rightarrow$ continue en 0

i.e. $\lim_{\Delta x \to 0} \cos \Delta x = \cos 0 = 1$

<u>DEM</u>:



$$\frac{\sin \Delta x}{\cos \Delta x} = \operatorname{tg} \Delta x$$

On remarque $\frac{\Delta x}{2} < \frac{\operatorname{tg} \Delta x}{2}$

$$\Leftrightarrow \quad \cos \Delta x < \frac{\sin \Delta x}{\Delta x} \underbrace{<}_{\text{par le dessin}} 1$$

Comme $\lim\limits_{\Delta x \to 0} \cos \Delta x = 1$

On a $\lim\limits_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} = 1$

<u>Conclusion</u>: $\sin x$ est différentiable en 0

et $\sin'(0) = 1$

$$\sin'(x_0) = \lim_{\Delta x \to 0} \frac{\sin(x_0 + \Delta x) - \sin(x_0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin(x_0) \cos \Delta x + \sin(\Delta x) \cos(x_0) - \sin(x_0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \cos(x_0) \underbrace{\frac{\sin(\Delta x)}{\Delta x}}_{\to 1} + \sin(x_0) \underbrace{\frac{(\cos \Delta x - 1)}{\Delta x}}_{\to 0}$$

$$\boxed{\sin'(x_0) = \cos(x_0)}$$

$$\cos'(x_0) = \lim_{\Delta x \to 0} \frac{\cos(x_0 + \Delta x) - \cos(x_0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x_0 \cos \Delta x - \sin x_0 \sin \Delta x - \cos x_0}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \cos x_0 \underbrace{\frac{(\cos \Delta x - 1)}{\Delta x}}_{\to 0} - \sin x_0 \underbrace{\frac{\sin \Delta x}{\Delta x}}_{\to 1}$$

$$\boxed{\cos'(x_0) = -\sin x_0}$$