MAT5361/4162 Midterm overview

October 29, 2007

General information

The midterm will consist of some multiple choice questions and some long answer questions. The long answer questions will test your knowledge of concepts and results as well as your ability to apply them. See below for sample questions to get an idea about what your will be expected to do.

Material

- Chapters 1,2,3 in the notes: everything except for the sections marked with a *, and except for 3.1.3.
- Chapter 4 in the notes: sections 4.1.1 and 4.1.2.

If you're studying from the book, you need to do

- Chapter 1
- Chapter 2, but not 5.5, 5.6
- Chapter 3: only sections 2 and 3
- Chapter 4,5
- Chapter 6 section 1
- Chapter 7 sections 1,2 (omit Theorem 2.16) and 3
- Chapter 9 sections 1,2,3
- Chapter 11 (omit discussion about recursive operators)

Sample problems

1. From a predicate $P(x)$ and a function $f(x)$ we may construct a function h as
$$h(x) = \mu z \leq f(x).P(z)$$
Thus $h(x)$ is the least $z \leq f(x)$ s.t $P(z)$ is true, and is 0 otherwise. Show that if P and f are primitive recursive, so is h.

2. Let f be a recursive function such that $dom(f) = \{x | x \text{ is odd}\}$. Consider the function
$$h(x) = \mu z.z > f(x).$$
Is $h(x)$ recursive? What is $h(0)$?

3. Write a register machine program which computes the function
$$f(x) = \begin{cases} x & \text{if } x \text{ is even} \\ x+1 & \text{if } x \text{ is odd.} \end{cases}$$

4. Give the definition of an index set and state Rice's Theorem. Decide whether the following two sets are recursive or not: the set
$$A = \{x | \text{ the program with code } x \text{ has less than } x \text{ instructions}\}$$
$$B = \{x | \text{ the program with code } x \text{ halts on input } 12\}$$

5. Explain in less than 100 words why the computable functions are exactly the recursive functions.

6. State the Parameter theorem, and explain how we can use it to define a total recursive function f such that
$$f(x) \bullet y = \phi_{x+1}(y).$$

7. State the second recursion theorem and use it to prove that there exists a code e such that
$$e \bullet x = \phi_e(e).$$

8. Prove that the Halting set $K = \{x | \phi_x(x)\downarrow\}$ is not recursive.

9. Give the definition of an m-complete set. Prove that the Halting set is m-complete.

10. Show that the set $A = \{x | \phi_x(x) = 0\}$ is m-complete. Use this to prove that there exists a countable family of disjoint m-complete sets.

11. Show that if $A \equiv_m B$ then $\mathbb{N} - A \equiv_m \mathbb{N} - B$.

12. Prove or disprove: $\mathsf{Empty} \equiv_m K_1$.

13. Prove that the set $\mathsf{Tot} = \{x | \phi_x \text{ is total}\}$ is not r.e. Hint: use a diagonal argument.

14. Explain what is meant by an enumeration of a given class of functions. Does there exist an enumeration theorem for the class of primitive recursive functions? Why (not)?

15. Let D_y denote the finite set with code y (assuming we have a primitive recursive bijection between finite subsets and natural numbers). Write $\sharp(E)$ for the code of the finite set E. Prove that there exists a primitive recursive function f such that

$$f(y) = \sharp(D_y \cup \{y\}).$$

16. Let $S(e, x)$ be the function

$$S(e,x) = \begin{cases} y & \text{if } T(e,x,y) \text{ for some } y \\ 0 & \text{otherwise.} \end{cases}$$

Is the function S recursive?