

MAT5361: Recursion Theory

Overview and aims of the course. In this course we start with an introduction to basic recursion theory, traditionally regarded as one of the four main pillars of mathematical logic, but also regarded as being strongly interwoven with theoretical computer science. After covering the basic theory of computable functions, computable sets and recursively enumerable sets, we discuss various applications and advanced topics, depending on student interest.

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course homepage:

http://www.mathstat.uottawa.ca/~phofstra/teaching/5361_07.htm

Course contents. Recursion theory is the study of functions on, and subsets of, the natural numbers. At an elementary level, it is concerned with the question which such functions are computable by algorithmic means. (For this reason, the term “computability theory” is used interchangeably with “recursion theory”.) Therefore, we will start by studying the class of computable functions and by giving various characterizations of this class, for example in terms of Turing machines. Among the results we obtain are: parametrization theorem, various forms of the recursion theorem, the Normal form theorem, and the universality theorem.

One step more complicated are the recursively enumerable (r.e.) sets; these are sets for which membership is not decidable, but whose elements can still be generated by an algorithmic procedure. We establish the basic results, in particular various characterizations of r.e. sets, inseparability results, Rice’s theorem, but also look at some interesting applications of the theory in logic and other areas of mathematics. For example, r.e. sets play a key role in the solution to Hilbert’s tenth problem (about Diophantine equations) and also figure in combinatorial group theory.

Next, we look at various notions of reduction, which allows us to compare functions and sets w.r.t. to their level of difficulty; this gives rise to degree theory, a complicated area with many deep results. We will also study the arithmetic hierarchy, which not only provides us with a language for classifying certain functions, but also brings about an interesting connection with the theory of Peano Arithmetic.

The above material will form the core of the course. From there, we can look at some advanced topics, depending on student interest. Here are just a few of the possible options: connections with topology (theorems by Myhill-Shepherdson, Rice-Shapiro and Kreisel-Lacombe-Shoenfield); degree theory (in particular priority techniques, and the solution to Post’s problem); higher recursion theory (notions of functionals of higher type); abstract formulations of recursion theory (Partial Combinatory Algebras, lambda calculus, category theory); Gödel’s Incompleteness Theorems.

Prerequisites. This course is relatively self-contained, but requires mathematical maturity; also, some background in logic (especially first order predicate

logic, for example MAT3361) is desirable. The course is especially recommended to students with an interest in logic or theoretical computer science. If you are in doubt whether this course is appropriate for you, do not hesitate to contact your instructor.

Course details.

- Classes: Mon. 8.30-10.00am, KED B005, Wed. 8.30-10.00am, KED B015.
- First class: Sept. 10th.
- Marking scheme: there will be a midterm exam worth 20%, a scheduled final exam worth 50%, homework assignments worth 15% and a project assignment worth 15%.

Literature. Cutland, N.J.: “Computability - An introduction to recursive function theory”, Cambridge University Press 1980. Where needed, additional material will be made available.