Category Theory Octoberfest 2015 Abstracts

Ettore Aldrovandi: *Stacks of categorical rings and their morphisms*
Abstract:

Michael Barr: *Limit closures of some full subcategories*
Abstract: John Kennison, Bob Raphael, and I have recently been working on the question of the limit closure of a full subcategory of a complete category inside that larger category. I will report on the results of these investigations in three cases:

(1) The subcategory of metric spaces inside the category of separated uniform spaces.
(2) The subcategory of integrally closed domains inside the category of commutative rings.
(3) The subcategory of all integral domains inside the category of commutative rings.

Bill Boshuck: *The truncated presentation of closed multicategories*
Abstract:

Marta Bunge: *Synthetic Theory of Stable Mappings and their Singularities*
Abstract: Available at http://mysite.science.uottawa.ca/phofstra/Oct2015/abstract-bunge.pdf

Robin Cockett: *The basics of integral categories*
Abstract: Differential categories of both tensor and Cartesian stripes have been around for quite a while: what do the corresponding structures for integral categories look like? Some answers and some models ... (This is joint work with Bauer, Blute, Lemay, ...)

Jeff Egger: *On the social life of generalised Hilbert objects*
Abstract: Involutive monoidal categories (IMC's) provide a nice framework to discuss generalised Hilbert objects (GHO's); in this talk I will show how to pass GHO's from one IMC to another – which is trickier than it ought to be!

Brendan Fong: *Decorated cospans, corelations, and electrical circuits*
Abstract: Given a category $\mathcal{C}$ with finite limits, it is well-known that one may form a category with morphisms spans in $\mathcal{C}$, and composition given by pullback. Furthermore, when an epi-mono factorisation system is available, one may form a category with morphisms jointly-monic spans in $\mathcal{C}$. In the category **Set** this construction gives relations between sets. Dually, one may talk of categories of cospans and corelations.
Given a cospan or corelation $X \to N \leftarrow Y$, we may equip the so-called apex N of the cospan with additional structure, such as a finite graph with vertices N, or a real valued function on N. We call this additional structure a decoration, and the cospan a decorated cospan. We shall discuss the

conditions under which this construction gives a category, and how to construct functors between such categories.
As an example, we shall show that a class of electrical circuit diagrams can be considered as morphisms in a decorated cospan category, and that the semantics of these circuit diagrams may be view as a functor from this category to the category of linear relations.
This is joint work with John Baez (UC Riverside).

Peter Freyd: *Calculus equationally*
Abstract: The existence of injective extensions in its category of models strongly limits an equational theory; in particular, it prohibits the possibility of a "compactness theorem" for solving sets of equations and that prohibition has limited the usefulness of nonstandard analysis. (Given, for example, a model of the reals and a set of equations on a variable x of the form $\max\{c, x\} = x$, one for each element c, any finite set of such equations has a solution. In any injective model there would have to be a simultaneous solution for all of them.)
By switching from Euclidean spaces to cubes we avoid the problem and a remarkably large part of traditional analysis can be made entirely equational. Yes, if we stick to maps between intervals we can characterize continuous derivatives without recourse to limits. (And we can easily move to maps between n-cubes.) Going in a different direction, we can deliver – in each dimension – the family of Lebesgue-measurable functions on a cube as the injective envelope of an easily (categorically easily) identified object.

Sakif Khan: *Isotropy and Covering Toposes*
Abstract: A well-known result in the theory of Grothendieck toposes states that any topos $\mathcal{E}$ admits a cover $\mathrm{Sh}(X) \twoheadrightarrow \mathcal{E}$, where X is a locale known as the Diaconescu cover of $\mathcal{E}$. However, the localic cover thus obtained can vary wildly with the site chosen for $\mathcal{E}$. In recent work of Funk, Hofstra and Steinberg, it is shown that each Grothendieck topos contains inside it a canonical group object called the isotropy group. The isotropy group of a topos encodes algebraic data in much the same way the subobject classifier carries spatial data and the notion of isotropy allows demarcation of Grothendieck toposes into two classes: those with isotropy and those without (anisotropic toposes). It is hoped that by weakening localic to anistropic, we may obtain a better class of covers for toposes. We report on some progress in this direction. Along the way, we shall introduce our parallel to the Diaconescu cover and briefly discuss how it may be realised as a kind of universal completion similar to the Karoubi envelope.

Hongliang Lai: *Flat weights vs forward Cauchy nets in categories enriched in a quantale*
Abstract: Categories enriched in a quantale are treated as quatitative preordered sets in quantitative domain theory. To investigated the directed completeness of quantale enriched categories, both the flat weights and the forward Cauchy nets are introduce in the literature. We show that if the quantale is the unit interval equipped with a continuous t-norm, flat weights are equivalent to forward Cauchy nets if and only if the quantale has non-trivial idempotent elements. (It is based on a joint-work with Wei Li and Dexue Zhang, in preprint).

Weiyun Lu: *Many-valued algebraic logic - MV and effect algebras*
Abstract: Introduced by C.C. Chang in the 1950s, MV algebras are to many-valued (Łukasiewicz) logics what boolean algebras are to two-valued logic. We begin by discussing MV algebras and some motivation to the study of many-valued logic. We then introduce effect algebras, a class of partial algebras recently introduced by mathematical physicists to describe quantum effects. One

can then say that what MV algebras are to classical many-valued logics, effect algebras are to quantum many valued logics - in fact, it was discovered that effect algebras are generalizations of MV algebras (though the references to and proofs given of this statement in the present literature are a mess)! We investigate some categorical structure of effect algebras, in particular how partiality causes coequalizers to be complicated, and characterize the regular monomorphisms.

Rory Lucyshyn-Wright: *Relative symmetric monoidal closed categories I: Autoenrichment and change of base*
Abstract: Every symmetric monoidal closed category $\mathcal{V}$ underlies an associated enriched category, but in what sense is this assignment functorial? To begin, we answer this question by showing that this assignment extends to a 2-functor valued in an op-2-fibred 2-category of symmetric monoidal closed categories enriched over various bases. Next, for a fixed $\mathcal{V}$, we show that this induces a 2-functorial passage from symmetric monoidal closed categories over $\mathcal{V}$ (i.e., equipped with a morphism to $\mathcal{V}$) to symmetric monoidal closed $\mathcal{V}$-categories over $\mathcal{V}$. We outline how this in turn leads to several equivalences of 2-categories that compare distinct notions of relative symmetric monoidal closed category, thus establishing several elaborations and variations on an unpublished result of G. M. Kelly. We discuss a rich source of examples in algebraic geometry, namely the ordinary and derived direct image and inverse image functors induced by morphisms of schemes.

Gábor Lukács: *Algebra in the category of weakly complete spaces: the character groups of Hopf algebras*
Abstract: (Joint with Rafael Dahmen.) For a Hopf algebra H (without a topology) over a topological field $\mathbb{K}$, the unital algebra homomorphisms $H \to \mathbb{K}$ equipped with the pointwise topology form a topological group. The *inverse problem* calls for characterizing all topological groups of this form.
A topological vector space over $\mathbb{K}$ is *weakly complete* if it is isomorphic to $\mathbb{K}^J$ for a set J. The category $\mathsf{WC}_{\mathbb{K}}$ of weakly complete spaces over $\mathbb{K}$ and their continuous homomorphisms admits a monoidal structure, and is equivalent to $\mathsf{Vect}_{\mathbb{K}}^{\mathsf{op}}$ with the usual tensor product.
In this talk, we present results concerning algebraic structures in the category $\mathsf{WC}_{\mathbb{K}}$ with the view to providing a partial answer to the inverse problem.
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Michael Makkai: *Closed multi-categories*
Abstract: This is joint work with Bill Boshuck. Closed multicategories – (possibly) without tensor product! – was shown by Manziuk and his colleages (TAC, 2013) are "the same" as Eilenberg-Kelly closed categories (with a slight change in the E-K definition). In particular, the multi-structure is reproducible from the Eilenberg-Kelly type of (rather complicated) closed structure. However, the multi-structure is useful and easy to work with. "Closed" is defined in the presence of the multi-structure more simply than in the E-K work; think of Abelian groups in the multi-linear context. We prove the theorem that four-truncated closed multicategories) are all you need: the higher arities can be reconstructed. Our aim is to give applications in higher-dimensional category theory: bicategories, Gray categories, Crans teisi (higher semi-strict categories). I (MM) was talking about related matters in Fredericton and Halifax in 2010, but matters are more advanced now. The main point is that one does not need the existence of the tensor product; the multi-structure imitates to a sufficient degree in our examples the work done by the tensor.

Susan Niefield: *Exponentiable Locale Morphisms*

Abstract: It is well known that a morphism $Y \to B$ of locales is exponentiable if and only if the corresponding internal locale is locally compact in the topos of sheaves on B. In this talk we present a completely external characterization and proof of this result.

Keith O'Neill: *Hochschild Homology in Codifferential Categories*
Abstract: Codifferential categories arise as dual categories to models of differential linear logic. This talk will serve to introduce codifferential categories to a general audience as well as to elucidate their application. In particular, it will be shown that in this context one may frame important questions about smoothness in the sense of algebraic geometry much more abstractly and in such a way that facilitates a new approach to calculations of invariants. Fundamental to this work is the interaction between Kahler n-forms for algebras and the algebra modalities of codifferential categories; exploiting this relationship allows one to, for instance, phrase the Hochschild-Kostant-Rosenberg theorem in a way that does not presuppose commutativity.

Bob Paré: *Superspans*
Abstract: I will discuss a slight generalization of the span construction based on a category with a "larger" category (super category) which has a choice of pullbacks. This simple idea leads to nice examples of lax or colax double functors which are not pseudo nor even normal. Time permitting, I will show how this gives good examples of intercategories in which the interchangers are non-trivial.

Jason Parker: *Duality between Cubes and Bipointed Sets*
Abstract: In light of the new model of intensional type theory in the category of cubical sets, we prove that an expanded version of the cube category (which we call the Cartesian cube category) is dual to the structurally simpler category of finite strictly bipointed sets. This entails that cubical sets (presheaves on the cube category) are equivalent to the structurally simpler covariant presheaves on such finite bipointed sets. This duality of base categories also allows us to show that the Cartesian category of cubes is the free finite product category on an interval. Finally, we show that the category of finite weakly bipointed sets is dual to the free finite limit category on an interval.

Dorette Pronk: *Orbifold Atlases for Nonreduced Orbifolds*
Abstract: Classical orbifolds (called V-manifolds at that time) were introduced by Satake in terms of an underlying space with an atlas of orbifold charts. Local charts were given in terms of an open subset of Euclidean space with an effective action of a finite group. Because of the effectiveness of the actions, embeddings between these charts could be taken to be all embeddings between the charts that commute with the group actions. However, the fact that not all structure present was really used became apparent in the definition of maps between orbifolds. Satake presented two different notions in his papers, and neither one of those was the right one for orbifold homotopy theory. This problem was remedied by considering orbifolds as a particular kind of (effective) Lie groupoids. In this description the natural notion of map would be that of a Hilsum Skandalis module or a so called generalized map. (Generalized maps are obtained as maps in the bicategory of fractions constructed to invert the Morita equivalences between Lie groupoids.)
In the study of orbifold homotopy theory and its applications to mathematical physics it became apparent that it would be desirable to study orbifolds for which the group actions on the charts are not necessarily effective. In terms of groupoids it was easy to drop the effective requirement and generally non-effective (also called, non-reduced) orbifolds have been studied in terms of their representing groupoids. Several authors do include a sketch of how they would consider these

objects as an underlying space with an atlas of (possibly non-effective) charts with a collection of embeddings that need to satisfy certain properties. However, none of those definitions provide a notion of orbifold that actually corresponds to the non-effective Lie groupoids.
In order to obtain this correspondence one needs to require additional structure on the collection of all embeddings between two charts and the non-effectiveness needs to be built right into the embedding structure. In this talk I will present a way to do this in terms of modules. This is joint work with Laura Scull, Alanod Sibih, and Matteo Tomassini.

Lili Shen: *Morphisms of distributors: Chu connections and back diagonals*
Abstract: Each category has its arrow category, which admits a natural quotient category of diagonals known as the Freyd completion of categories. For closed bicategories, dual constructions of its arrow category and Freyd completion may be formed as the category of Chu connections and that of back diagonals. This talk will present a brief introduction to these structures and their closed relations to generalized Chu constructions (in particular, Chu spans), morphisms of distributors, morphisms of closure spaces and formal concept analysis in computer science. Based on joint-work with Yuanye Tao and Dexue Zhang ("Chu connections and back diagonals between $\mathcal{Q}$-distributors", 2015).

Polina Vinogradova: *Formalizing abstract computability : Turing categories in Coq*
Abstract: Turing categories, developed by Robin Cockett and Pieter Hofstra, are an abstraction that captures the key features of recursion theory by means of categorical structure. Recursion can also be done using a formal language to express the relevant structure. This talk will explain how the formal language Calculus of (co-)Inductive Constructions, underlying the proof assistant Coq, can be used to capture the more abstract categorical way of doing recursion theory, leading to developing a more rigorous, structured and verifiably correct approach to studying abstract computability.