MATH 1300-MIDTERM # 2-2009

NAME and I.D.#_____

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 60 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages. Your answers for numerical questions should be ready to be plugged into a calculator (i.e. you may leave an answer such as $x = \frac{2}{3 \ln 4}$ without simplifying).

For long answer questions, YOU MUST SHOW YOUR WORK

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

ANSWERS:

☐	☐	☐	☐
#1	#2	#3	#4

Multiple Choice Section Questions (1-4)

Question 1 *Find all asymptotes of the function* $f(x) = \frac{x^2+3x-4}{x^2+6x+8}$.

A) vertical asymptote at $x = -2$, no horizontal asymptotes
B) vertical asymptotes at $x = -2, x = -4$, no horizontal asymptotes
C) vertical asymptote at $x = -4$, horizontal asymptote at $y = 0$
D) vertical asymptote at $x = -2$, horizontal asymptote at $y = 1$
E) vertical asymptotes at $x = -2, x = -4$, horizontal asymptote at $y = 0$
F) vertical asymptotes at $x = -4, x = -2$, horizontal asymptote at $y = 1$

$f(x) = \frac{(x-1)(x+4)}{(x+2)(x+4)}$

V.A. at $x = -2$

H.A. at $y = 1$

Question 2 *Suppose that the demand function for a product is given by* $p = 8 + \frac{120}{x+5}$. *What is the elasticity of demand when* $x = 55$*? Is demand elastic or inelastic?*

A) $\eta = -\frac{10}{55}$, elastic B) $\eta = -\frac{10}{55}$, inelastic C) $\eta = -\frac{1}{25}$, elastic

D) $\eta = -\frac{1}{25}$, inelastic E) $\eta = -\frac{60}{11}$, elastic F) $\eta = -\frac{60}{11}$, inelastic

$p(55) = 8 + \frac{120}{60} = 10$ $p'(x) = -\frac{120}{(x+5)^2}$

$p'(55) = -\frac{120}{(60)^2} = -\frac{1}{30}$

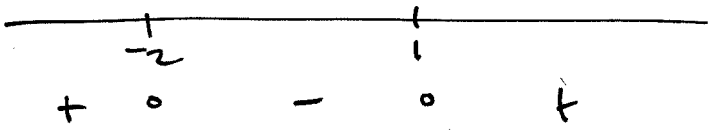
$\eta = \frac{p/x}{p'(x)} = \frac{10/55}{-1/302} = -\frac{300}{55} = -\frac{60}{11}$,

elastic

Question 3 *Given the function* $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 2$, *which of the following statements is correct?*

A) $f(x)$ has a local minimum at $x = -2$
B) $f(x)$ has a local maximum at $x = -2$
C) $f(x)$ has an inflection point at $x = -2$
D) $f(x)$ has a local maximum at $x = -\frac{1}{3}$
E) $f(x)$ has a local minimum at $x = -\frac{1}{3}$
F) $f(x)$ has an inflection point at $x = -\frac{1}{3}$

$f'(x) = x^2 + x - 2 = (x-1)(x+2)$ so



By 1st der. test $x=-2$ is a local max.

Question 4 *Consider the function* $g(x) = x^3 - \frac{3}{2}x^2 - 6x$. *Where does the absolute minimum of* $g(x)$ *on the interval* $[-4, 4]$ *occur?*

A) At $x = -4$ **B)** At $x = -1$ **C)** At $x = 0$ **D)** At $x = 2$ **E)** At $x = 4$

$g'(x) = 3x^2 - 3x - 6 = 3(x-2)(x+1)$

$g(-4) = -64$
$g(-1) = 4\frac{1}{2}$
$g(2) = -10$
$g(4) = 12$

$\Rightarrow$ abs. min occurs at $x = -4$

Long Answer Section Questions (5-7)

Question 5 (12 points) *A frog population on a tropical island is growing exponentially. In 1999, there were 200 animals, and in 2005 there were 240 animals.*

(a) (6 points) Find a formula which describes the size of the frog population as a function of time (measured in years).

(b) (2 points) What will the population size be in 2024?

(c) (4 points) How many years will it take before there are 600 frogs on the island?

(a) $P(t) = P_0 e^{kt}$ $\quad P_0 = 200$ $\quad P(6) = 240$

$240 = 200 \cdot e^{k \cdot 6} \Rightarrow e^{6k} = 1.2$

$\Rightarrow 6k = \ln(1.2)$

$\Rightarrow k = \frac{\ln(1.2)}{6}$

$\Rightarrow \underline{P(t) = 200 \cdot e^{\ln(1.2)/6 \cdot t}}$

(b) $P(25) = 200 \cdot e^{\ln(1.2)/6 \cdot 25}$.

(c) $P(t) = 600 \iff 600 = 200\, e^{\ln(1.2)/6 \cdot t}$

$\iff e^{\ln(1.2)/6 \cdot t} = 3$

$\iff \ln(1.2)/6 \cdot t = \ln 3$

$\iff t = \frac{6 \ln(3)}{\ln(1.2)}$.

Question 6 (14 points) *Consider the function* $f(x) = -3x^{\frac{2}{3}} + 2x + 2$.

(a) (2 points) Find the derivative of f.

(b) (3 points) Find all critical points of this function.

(c) (4 points) Classify the critical points. Indicate which test(s) you use and how.

(d) (5 points) Find all inflection points and intervals where the function is concave up and all intervals where it is concave down.

(a) $f'(x) = -2x^{-1/3} + 2 = \frac{-2}{\sqrt[3]{x}} + 2$

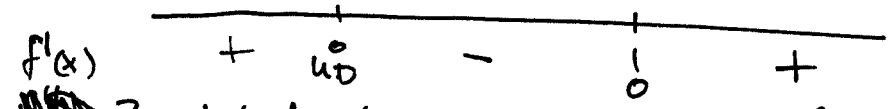
(b) $f'(x)$ is undefined at $x = 0$ ($f(0)$ is defined!)

so 0 is a C.P.

$f'(x) = 0 \iff \frac{2}{\sqrt[3]{x}} = 2 \iff \sqrt[3]{x} = 1$

$\iff \underline{x = 1}$ is a C.P.

(c)



By 1st der. test $x=0$ is a local max and $x=1$ a local min.

(d) $f''(x) = \frac{2}{3} \cdot \frac{1}{x\sqrt[3]{x}}$ undefined at $x = 0$.

f is concave up on $(-\infty, 0)$ and $(0, \infty)$ and has no inflection points.

Question 7 (14 points) *A company produces household appliances, and has found that the demand function for refrigerators is given by* $p = 200 + \frac{60{,}000}{\sqrt{x}}$. *The company has an initial cost of 10,000 dollars, and each unit costs 500 dollars to make. How many refrigerators should the company produce in order to maximize profit? Be sure to explain why your answer is an absolute maximum.*

$C(x) = 10000 + 500x \qquad R(x) = x \cdot p = 200x + 60000\sqrt{x}$

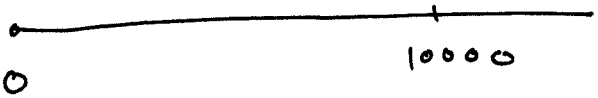
$P(x) = R(x) - C(x) = 60000\sqrt{x} - 300x - 10000$.

$P'(x) = \frac{30000}{\sqrt{x}} - 300$

$P'(x) = 0 \iff \frac{30000}{\sqrt{x}} - 300 \iff \sqrt{x} = 100$

$\iff x = \pm 10000$

Discard $x = -10000$; we have



0

$P'(x)$ ∄ + 0 −

By the fact that $x = 10000$ is the only CP on $(0, \infty)$ this is an abs. max.

Bonus Question (1 point): What do Dutch people traditionally put on their french fries?

Mayo

(certainly no vinegar!)

Space for additional work