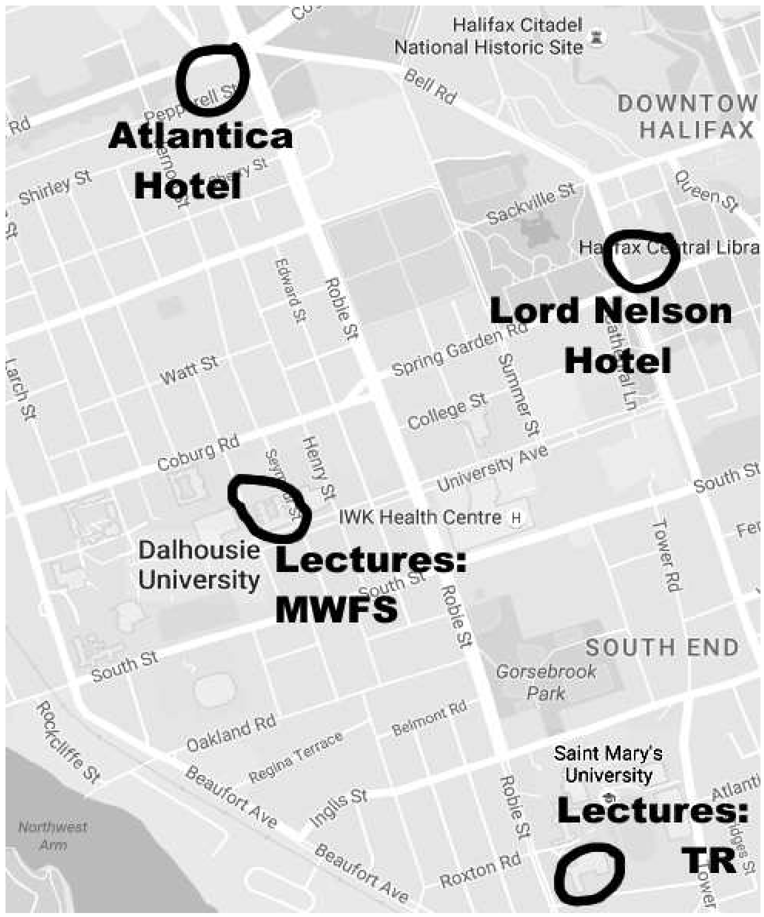
# Our Sponsors

AARMS was founded in March 1996 at a time when the National Network for Research in the Mathematical Sciences was being discussed and planned. AARMS exists to encourage and advance research in mathematics, statistics, computer science, and mathematical sciences, in the Atlantic region. In addition, AARMS acts as a regional voice in discussions of the mathematical sciences on a national level.
We would like to thank AARMS for support for CT16, and for supporting the summer school which triggered the idea of a conference in Halifax this summer.

Dalhousie University is Nova Scotia's largest university. Founded in 1818, it now offers a full range of undergraduate and graduate degrees. It has been a center of excellence in category theory for almost half a century.
We would like to thank the president's office, the dean of science, and the department of mathematics and statistics for their generaous support of CT16, and to give special thanks to Queena Crooker-Smith and Tianshu Huang for administrative assistance.

Saint Mary's University is Nova Scotia's second-largest university. It was founded in 1802 when the Reverend Edmund Burke, later Bishop Burke, taught young men at the Glebe House, on the corner of Spring Garden Road and Barrington Street. It is now a coeducational university with strong program in arts, science,and commerce.
We would like to thank the president, vice-president (academic and research) and the dean of science for their generous support of CT16.

# CT2016 Program




**Sunday August 7 Welcome Reception**
**(19:00 – 21:00) Lord Nelson Hotel**

# Monday August 8 - McCain Building, Dalhousie

| **Time** | **Speaker** |
|---|---|
| 08:45 – 09:00 | Welcome |
| 09:00 – 10:00 | John Bourke (Masaryk University)<br>*Grothendieck $\infty$-groupoids as iterated injectives* |
| 10:05 – 10.35 | Nick Gurski (University of Sheffield)<br>*2-linear algebra and the Picard group of Picard categories* |
| | Coffee Break |
| 11:00 – 11:30 | Alexander Campbell (Macquarie University)<br>*A higher categorical approach to Giraud's non-abelian cohomology* |
| 11:35 – 12:05 | David Spivak (MIT)<br>*String diagrams for traced and compact categories are oriented 1-cobordsisms* |
| | Lunch Break |
| 13:45 – 14:15 | Martti Karvonen (University of Edinburgh)<br>*Dagger category theory: monads and limits* |
| 14:20 – 14:50 | Roald Koudenburg (Middle East Technical University)<br>*A categorical approach to the maximum theorem* |
| | Tea Break |
| 15:20 – 15:50 | Toshiki Kataoka (University of Tokyo)<br>*Categories of Filters as Fibered Completions* |
| 15:55 – 16:25 | Ramón Abud Alcalá (Macquarie University)<br>*Lax Actions and Skew Monoids* |
| 16:30 – 17:00 | Rémy Tuyéras (Macquarie University)<br>*Sketches in Higher Category Theories and the Homotopy Hypothesis* |
| 19:30 | Public Lecture, Jamie Vicary: *Our Quantum Future*<br>Reception to follow! |

# Tuesday August 9 – Sobey Building, Saint Mary's

| **Time** | **Speaker** |
|---|---|
| 08:45 – 0900 | Welcome |
| 09:00 – 10:00 | Nicola Gambino (University of Leeds)<br>*Algebraic models of homotopy type theory* |
| 10:05 – 10.35 | Peter Lumsdaine (Stockholm University)<br>*Equivalences of (intensional) type theories and their models* |

Coffee Break

| | |
|---|---|
| 11:00 – 11:30 | Matias Menni (Universidad Nacional de la Plata)<br>*The construction of* $\pi_0$ *in the context of Axiomatic Cohesion* |
| 11:35 – 12:05 | Dimitris Tsementzis (Princeton University)<br>*Homotopy Model Theory* [note change!] |

Lunch Break

| | **Session 1** | **Session 2 (in Sobey 260)** |
|---|---|---|
| 13:45 – 14:10 | Tim Campion<br>*Locally Presentable Categories of Topological Spaces* | |
| 14:15 – 14:40 | Jonathan Gallagher<br>*Categorical models of differential logic* | |
| 14:45 – 15:10 | Jeff Egger<br>*On open locales and inner products* | |

Tea Break

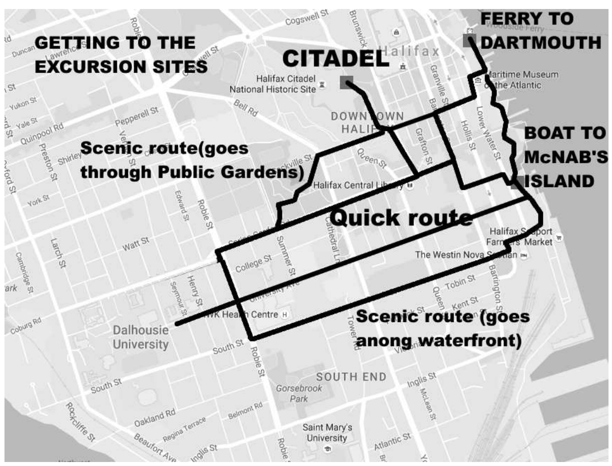
| | | |
|---|---|---|
| 15:35 – 16:00 | Spencer Breiner<br>*Categories for the working engineer* | Branko Nikolić<br>*Strictification tensor products* |
| 16:05 – 16:30 | Salil Samant<br>*Fibred Signal Representation* | Alanod Sibih<br>*A New Notion of Orbifold Atlas* |
| 16:35 – 17:00 | Yoshihiro Maruyama<br>*Categorical Foundations for Big Data* | |

**LUNCH: Tuesday and Thursday:** Saint Mary's is in a largely-residential neighborhood and does not have a lot of restaurants nearby. You can eat on campus: go down the stairs *and ramps* outside the Conference Theater, then walk straight, with a leftward jog, to the Loyola Colonnade. If you're not very hungry, try Tim Horton's on your left. Turning right, walking through the first doors, left, and right at the residence desk brings you to the Dockside Dining Hall for a more solid and balanced meal.

Off campus: If you walk fast, there might be time to get to Bonehead's BBQ at Inglis and Barrington, or even the Granite Brewery a few blocks north of there. But the Dockside would be a safer bet.

## Wednesday August 10 – McCain Building, Dalhousie

| Time | Speaker |
|---|---|
| 09:00 – 10:00 | Dominic Verity (Macquarie University)<br>*On The Yoga of (Un)Straightening* |
| 10:05 – 10.35 | Emily Riehl (Johns Hopkins University)<br>*Higher order chain rules for abelian functor calculus* |
| | Coffee Break |
| 11:00 – 11:30 | Jamie Vicary (Oxford University)<br>*Semistrict 4-categories and strong adjunctions* |
| 11:35 – 12:05 | Pierre Cagne (Ecole Normale Supérieure)<br>*Bifibrations of model categories and the Reedy construction* |
| 12:10 – 12:40 | Daniel Christensen (University of Western Ontario)<br>*Higher Toda brackets and the Adams spectral sequence* |




On Wednesday afternoon there are three choices for excursions. These are covered in registration fees for conference participants.
(A) Halifax Citadel: a historical site with guides and reenactors. Accompanying persons will pay about $10 entrance fee.
(B) Taking the ferry to Dartmouth, walking on a pleasant trail to the Woodside terminal, and taking the ferry back. Cost to accompanying persons, $2.50 for the ferry ticket (get a transfer for the way back!) and the cost of your coffee and snack.
(C) McNab's Island. Note that there are two sailings because the capacity of the boat is limited. Please go at the time you sign up for (sheets will be available ahead of time). Accompanying persons are asked to pay $23 per person to Robert Dawson or Damien DeWolf before boarding.

## Thursday August 11 – Sobey Building, Saint Mary's

| **Time** | **Speaker** |
|---|---|
| 09:00 – 10:00 | Catharina Stroppel (University of Bonn)<br>*Fractional Euler characteristics and why should we care?* |
| 10:05 – 10.35 | Clemens Berger (Université de Nice - Sophia Antipolis)<br>*Hyperplane arrangements, graphic monoids and moment categories* |

Coffee Break

| | |
|---|---|
| 11:00 – 11:30 | Richard Blute (University of Ottawa)<br>*The Shuffle Quasimonad and Modules with Differentiation and Integration* |
| 11:35 – 12:05 | Rory Lucyshyn-Wright (Mount Allison University)<br>*Enriched algebraic theories,monads, and commutants* |

Lunch Break

| | **Session 1** | **Session 2** |
|---|---|---|
| 13:45 – 14:10 | Darien DeWolf<br>*Restriction monads* | Jun Yoshida<br>*Cobordisms with strings* |
| 14:15 – 14:40 | Hongliang Lai<br>*Quantale-valued Approach Spaces* | Yuki Kato<br>*Motivic derived algebraic geometry* |
| 14:45 – 15:10 | Keith O'Neill<br>*Smoothness in Differential Categories* | Yuyi Nishimura<br>*Span equivalence between weak n-cat's* |

Tea Break

| | | |
|---|---|---|
| 15:35 – 16:00 | Ben MacAdam<br>*Partial Fermat theories* | |
| 16:05 – 16:30 | Kadir Emir<br>*2-Crossed Modules of Hopf Algebras* | Christian Espíndola<br>*Infinitary first-order categorical logic* |
| 16:35 – 17:00 | Chad Nester<br>*Turing Categories and Realizability* | Lucius Schoenbaum<br>*A Generalized Globular Approach to (Parts Of) Higher Topos Theory* |

Thursday evening is the conference banquet, at the Shore Club, Hubbards. Buses will be provided, leaving at 18:15 from Seymour Street (the street immediately to the east of the McCain building, where we met on Monday and Wednesday.) If you're registered with the conference,your ticket is covered. Anybody else must buy a ticket on Eventbrite *by Monday.* Please let the organizers know if you want the chicken or vegetarian option (lots of lobster for everybody else!)

## Friday August 12 – McCain Building, Dalhousie

| Time | Speaker |
|---|---|
| 09:00 – 10:00 | André Joyal (Université de Quebec à Montréal)<br>*A classifying topos for Penrose tilings* |
| 10:05 – 10.35 | Ross Street (Macquarie University)<br>*Cherchez la Cogèbre* |
| | Coffee Break |
| 11:00 – 11:30 | Walter Tholen (York University)<br>*Syntax and Sematics of Monad-Quantaloid Enrichment* |
| 11:35 – 12:05 | Susan Niefield (Union College)<br>*Coexponentiability and Projectivity: Rigs, Rings, and Quantales* |
| | Lunch Break |
| 13:45 – 14:15 | Jonathon Funk (CUNY - Queensborough)<br>*The centralizer of a topos* |
| 14:20 – 14:50 | Sakif Khan (University of Ottawa)<br>*Computing Isotropy in Grothendieck Toposes* |
| | Tea Break |
| 15:20 – 15:50 | Kris Chambers (University of Ottawa)<br>*The Isotropy Group of Continuous G-sets* |
| 15:55 – 16:25 | Jean-Simon Lemay (University of Calgary)<br>*Introduction to Cartesian Integral Categories* |
| 16:30 – 17:00 | Clive Newstead (Carnegie Mellon University)<br>*Categories of Natural Models of Type Theory* |

## Saturday August 13 – McCain Building, Dalhousie

| Time | Speaker |
|---|---|
| 09:00 – 10:00 | Dorette Pronk (Dalhousie University)<br>*Mapping Spaces for Orbispaces* |
| 10:05 – 10.35 | Geoff Cruttwell (Mount Allison University)<br>*A simplicial foundation for de Rham cohomology in a tangent category* |
| | Coffee Break |
| 11:00 – 11:30 | Robin Cockett (University of Calgary)<br>*Connections in Tangent Categories* |
| 11:35 – 12:05 | Howard Marcum (Ohio State University)<br>*Tertiary homotopy operations in a 2-category* |
| 12:10 – 12:40 | Robert Paré (Dalhousie University)<br>*Isotropic Intercategories* |

# Abstracts

## Lax Actions and Skew Monoids

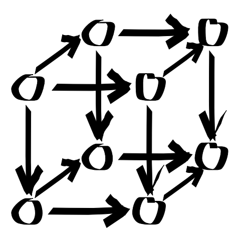
Ramón Abud Alcalá

In a monoidal bicategory $\mathcal{B}$, let $R$ be an object with a left adjoint $i : I \to R$, and a biduality $R \dashv R^o$. It was shown in [2] that, under certain conditions, opmonoidal monads on the monoidale $R^o \otimes R$ are in equivalent correspondence with right skew monoidales on $R$ that have $i$ as the unit. An adaptation of this theorem was presented in [1], for arrows that are not monads anymore, but only opmonoidal arrows of the sort $R^o \otimes R \to S^o \otimes S$, for suitable $R$ and $S$. These opmonoidal arrows are, under similar conditions as before, in equivalent correspondence with a relaxed version of an action, named right lax $R$-actions on $S$.

On one hand, it is clear, at least intuitively, that these two theorems are somehow related: for if one defines $\mathsf{Opmon}(\mathcal{B})$ to be the bicategory with monoidales, opmonoidal arrows and, opmonoidal cells in $\mathcal{B}$; then monads in $\mathsf{Opmon}(\mathcal{B})$ are opmonoidal monads in $\mathcal{B}$. But on the other hand, it is not clear how to define a "bicategory of lax actions" —in fact, there is no intuitive way to compose them— so that "monads of lax actions" are right skew monoidales in $\mathcal{B}$.

In this talk, I will make these statements precise. First, by providing the collection of right lax actions with a simplicial structure, called $\mathsf{LaxAct}(\mathcal{B})$. Second, by proving that (a suitable notion of) monads in $\mathsf{LaxAct}(\mathcal{B})$ are right skew monoidales whose unit has a right adjoint in $\mathcal{B}$. Finally, I will show that, under certain conditions, $\mathsf{LaxAct}(\mathcal{B})$ is the nerve of a bicategory, which in turn, is equivalent to a full subbicategory of $\mathsf{Opmon}(\mathcal{B})$, whose objects are the monoidales $R^o \otimes R$ induced by bidualities $R \dashv R^o$.

[1] Ramón Abud Alcalá. *Comodules for coalgebroids.* June 2015. Presented at the International Category Theory Conference, Aveiro, Portugal.
[2] Stephen Lack and Ross Street. *Skew monoidales, skew warpings and quantum categories.* pages 1-18, May 2012.



# Hyperplane arrangements, graphic monoids and moment categories

Clemens Berger

Hyperplane arrangements in Euclidean space occur in group theory (Coxeter arrangements) and in topology (Salvetti complex, Orlik-Solomon algebra). The hyperplanes of an arrangement $\mathcal{A}$ decompose the Euclidean space into facets which form a *monoid* $\mathcal{F}_{\mathcal{A}}$ such that for all $x, y \in \mathcal{F}_{\mathcal{A}}$ the relation $xyx = xy\,(*)$ holds. This algebraic structure recovers not only the partial ordering of the facets, but also the intersection lattice $\mathcal{L}_{\mathcal{A}}$ which is the universal commutative quotient of $\mathcal{F}_{\mathcal{A}}$.

Following Lawvere [1], monoids fulfilling $(*)$ will be called *graphic*. The main purpose of this talk is to promote a *categorification* of graphic monoids which we call a *moment category*. A moment structure on a category $\mathcal{M}$ consists in specifying for each object $A$ a set $m_A$ of idempotent endomorphisms, called *moments*, and for each morphism $f : A \to B$, a pushforward operation $f_* : m_A \to m_B$, subject to four axioms. These axioms imply that $m_A$ is a *graphic submonoid* of $\mathcal{M}(A, A)$ for each object $A$ of $\mathcal{M}$.

It turns out that a moment structure in which all $m_A$ are commutative is precisely a *corestriction structure* in the sense of Cockett-Lack [2]. In other words, a moment structure is a "skew" corestriction structure in much the same way as a graphic monoid is a "skew" lattice, cf. [3]. Surprisingly, a moment structure in which all moments *split* is completely determined by the existence of a certain split factorization system, which we call an *active/inert* factorization system.

Such an active/inert factorization system on $\mathcal{M}$ induces a notion of $\mathcal{M}$-*operad* for which the composition reflects the combinatorics of decomposing moments into elementary one's. As motivating examples may serve the following categories, which all carry canonical split moment structures: Segal's category $\Gamma$, the simplex category $\Delta$ and Joyal's higher cell categories $\Theta_n$. $\Gamma$-operads are symmetric operads, $\Delta$-operads are nonsymmetric operads, and $\Theta_n$-operads are Batanin's $(n-1)$-terminal $n$-operads.

For each split moment category $\mathcal{M}$, there is also an $\infty$-categorical version of $\mathcal{M}$-operad. For $\mathcal{M} = \Gamma, \Delta$ and $\Theta_n$ these $\infty$-categorical $\mathcal{M}$-operads have already been investigated in literature, cf. Lurie [4], Gepner-Haugseng [5], and Barwick [6].

[1] Lawvere, F.W., Linearization of graphic toposes via Coxeter groups, *J. Pure Appl. Algebra* **168** (2002), 425–436.
[2] Cockett, J.R.B. and Lack, S., Restriction categories I: categories of partial maps, *Theor. Comp. Sci* **270** (2002), 223–259.
[3] Schützenberger, M.-P., Sur certains treillis gauches, *C. R. Acad. Sci. Paris* **224** (1947), 776–778.
[4] Lurie, J., *Higher Algebra*, see http://www.math.harvard.edu/∼lurie/, last

update: march 2016.
[5] Gepner, D., and Haugseng, R., Enriched $\infty$-categories via nonsymmetric $\infty$-operads, *Adv. Math.* **279** (2015), 575–716.
[6] Barwick, C., *From operator categories to topological operads*, arXiv:1302.5756.

## The Shuffle Quasimonad and Modules with Differentiation and Integration

(joint work with M.Bagnol, R. Cockett, and J.S.Lemay)

Richard Blute

Differential linear logic and the corresponding categorical structure, differential categories, introduced the idea of differential structure associated to a (co)monad. Typically in settings such as algebraic geometry, one expresses differential structure for an algebra by having a module with a derivation, i.e. a map satisfying the Leibniz rule. In the monadic approach, we are able to continue to work with algebras and derivations, but the additional structure allows us to define other rules of the differential calculus for such modules; in particular one can define a monadic version of the chain rule as well as other basic identities.

In attempting to develop a similar theory of integral linear logic, we were led to consider the shuffle multiplication. This was shown by Guo and Keigher to be fundamental in the construction of the free Rota-Baxter algebra, the Rota-Baxter equation being the integral analogue of the Leibniz rule. This shuffle multiplication induces a quasimonad on the category of vector spaces. The notion of quasimonad, called r-unital monad by Wisbauer, is slightly weaker than that of monad, but is still sufficient to define a sensible notion of module with differentiation and integration. We demonstrate this quasimonad structure, show that its free modules have both differential and integral operators satisfying the Leibniz and Rota-Baxter rules and satisfy the fundamental theorems of calculus. We also discuss the construction of free modules with integration.

## Grothendieck $\infty$-groupoids as iterated injectives

John Bourke

At the beginning of his manuscript Pursuing Stacks, Grothendieck gave a somewhat informal definition of globular weak $\infty$-groupoid. Around 25 years later, this definition was brought to attention and simplified by Georges Maltsiniotis.
In the first part of this talk I will give an introduction to Grothendieck $\infty$-groupoids. In the second part, I will describe a new viewpoint on these structures – as iterated algebraic injectives. Building on work of Thomas Nikolaus on algebraically fibrant objects, I will give some applications of this point of view.

## Categories for the working engineer: a call for applied CT

(joint work with E. Subrahmaninan)

Spencer Breiner

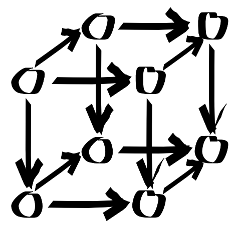
Applied category theory (CT) As a topic remains in its infancy. Well-accepted applications outside of mathematics are rare; those found in theoretical computer science and physics hardly count, as these topics are essentially mathematics themselves. This is not to say that there have not been applications studied in other areas (e.g., materials science [1]). but that these have been both piecemeal and largely ignored within their respective disciplines. Largely, it seems, we are speaking to the choir when we talk about applications of CT.

At the same time, there is real need for structural mathematics in commerce and industry. Millions of dollars and man-hours are spent each year dealing with ill-designed data formats and interchange difficulties. Often, these problems that could often be solved easily by thinking categorically, representing translations as functors. More generally, CT could provide a flexible vocabulary and a powerful, generic toolbox for working with information constructs. I will illustrate this with an example of applied CT from the realm of industrial scheduling.

The principle barrier to CT in real-world applications is this: the subject has a wicked learning curve. 50 years ago “abstract nonsense” was a problem for mathematicians, who are accustomed to thinking abstractly. How much worse, then, for engineers and scientists who just want tools to support their work. Thus the challenge of applied CT is two-fold: not only must we develop concrete applications using CT methods, we must also work out a methodology for teaching non-category theorists how to use them. This is a pedagogical problem as much as a technical one.

A solution require rethinking both how and when we teach CT. New CT learners need "intro level" presentations, where we give specific algorithms for solving specific problems. We must also integrate categorical methods into domain-specific presentations, so that experts from other fields can learn CT through resonant examples.

[1]Giesa, T., Spivak, D.I. and Buehler, M.J., *Advanced Engineering Materials*, **14.9**, 810-817 (2012), Wiley Online Library.



## Bifibrations of model categories and the Reedy construction

(joint work with P.-A. Melliès)

Pierre Cagne

In this talk, I will explain how to endow the total category $\mathcal{E}$ of a well-behaved Grothendieck bifibration $\mathcal{E} \to \mathcal{B}$ with a structure of a model category when both the basis $\mathcal{B}$ and all fibers $\mathcal{E}_b$ of the bifibration are model categories.

The motivating example is the well-known Reedy model structure on a diagram category $[\mathcal{R}, \mathcal{M}]$. The crucial step in its construction by transfinite induction lies in the successor case, which is usually handled by reasoning on latching and matching functors. A first observation is that those functors define a Grothendieck bifibration on the restriction functor $[\mathcal{R}_{<\lambda+1}, \mathcal{M}] \to [\mathcal{R}_{<\lambda}, \mathcal{M}]$ where $\mathcal{R}_{<\lambda}$ denotes the full subcategory of $\mathcal{R}$ whose objects have degree less than $\lambda$. Unfortunately, this bifibration fails to fulfil the conditions of application of the theorems appearing in [1] and [2], which would have allowed to lift the model structure from the base category $\mathcal{B} = [\mathcal{R}_{<\lambda}, \mathcal{M}]$ to the total category $\mathcal{E} = [\mathcal{R}_{<\lambda+1}, \mathcal{M}]$. The reason is that those theorems require that a cartesian lift to a fibrant object over a weak equivalence is acyclic in the model structure of $\mathcal{E}$, which is only true in the case of the Reedy construction when the category $\mathcal{M}$ is proper.

I will explain how to relax this hypothesis by focusing on acyclic fibrations and cofibrations and by only requiring that (co)cartesian lifts over acyclic (co)fibrations are acyclic (co)fibrations. This idea leads us to a simple and elegant condition for the construction: the commutative squares in the base category of the form $uv = v'u'$ with $u, u'$ acyclic cofibrations and $v, v'$ acyclic fibrations should satisfy a homotopical version of the Beck-Chevalley condition on the fibers. I will explain why the result generalizes [1] and [2], and illustrate how to apply it in order to obtain classical and generalized versions of the Reedy model structure theorem (cf.[3]).

[1] Stanculescu, A.E., Bifibrations and weak factorization systems,*Applied Categorical Structures*, 20(1):19-30, 2012
[2] Harpaz, Y, and Prasma, M., The Grothendieck construction for model categories, *Advances in Mathematics*, 218:1306-1363 (August 2015)
[3] Berger, C., and Moerdijk, I., On an extension of the notion of Reedy category, *Mathematische Zeitschrift* 269(3):977-1004, December 2011.

## A higher categorical approach to Giraud's non-abelian cohomology

Alexander Campbell

This talk continues the program of Ross Street and his collaborators to develop a theory of non-abelian cohomology with higher categories as the coefficient objects. The main goal of this talk is to show how this theory can be extended to recover Giraud's non-abelian cohomology of degree 2, thereby addressing an open problem posed by Street.

The definition of non-abelian cohomology that I adopt in this talk is one due to Grothendieck, which takes higher stacks as the coefficient objects; the cohomology is the higher category of global sections of the higher stack. I will compare these two approaches and show how they may be reconciled.

The central argument depends on a generalisation of Lawvere's construction for associated sheaves, which yields the 2-stack of gerbes over a topos as an associated 2-stack; the stack of liens (or "bands") is the 1-stack truncation thereof. I will also outline how the coherence theory of tricategories, supplemented by results of three-dimensional monad theory and enriched model category theory, provides a practicable model of the tricategory of 2-stacks over a site, in whose context the theory can be developed.

[1] Alexander Campbell. *A higher categorical approach to Giraud's non-abelian cohomology.* PhD thesis, Macquarie University, 2016.
[2] Jean Giraud. *Cohomologie non abélienne.* Springer-Verlag, Berlin-New York, 1971. Die Grundlehren der mathematischen Wissenschaften, Band 179.
[3] Alexander Grothendieck. *Pursuing stacks.* Typed notes, 1983.
[4] Ross Street. Categorical and combinatorial aspects of descent theory. *Appl. Categ. Structures*, 12(5-6):537–576, 2004.

## Locally Presentable Categories of Topological Spaces

Tim Campion

We study certain locally presentable subcategories of the category $\mathsf{Top}$ of topological spaces. If $\mathcal{C}$ is a small, full subcategory of the category $\mathsf{Top}$ of topological spaces, we show that the closure $\mathsf{Top}_{\mathcal{C}}$ of $\mathcal{C}$ under colimits in $\mathsf{Top}$ is locally presentable, and estimate the degree of accessibility using the theory of lax algebras for the ultrafilter monad. Using the axiomatization of [2], we also refine certain results of [1] by giving new estimates on the degree of accessibility of the category $\mathsf{Seq}$ of sequential spaces and the category $\mathsf{Top}_{\Delta}$ of delta-generated spaces, which we conjecture to be sharp.

[1] Lisbeth Fajstrup and Jiří Rosický. *A convenient category for directed homotopy*, TAC, 21(1):7–20, 2008.
[2] Gonçalo Gutierres and Dirk Hofmann, Axioms for sequential convergence, App. Cat. Structs., 15(5-6):599–614, 2007.

## The Isotropy Group of Continuous G-sets

Kris Chambers

Freyd's representation theorem [1] tells us that every Grothendieck topos $\mathcal{E}$ can can be obtained through a combination of: $Sh(\mathbb{B})$ sheaves over a boolean algebra, $\mathcal{B}(G)$ continuous $G$-sets for a topological group $G$, or, what Freyd referred to as, exponential varieties. Moreover, in the second case, it was shown that it is sufficient to take $G$ to be the automorphism group of an infinitely countable set with the topology generated by the pointwise stabilizers of finite sets.

The notion of Isotropy for a Grothendieck topos was developed by Hofstra, Funk and Steinberg in [2]. The isotropy group is an invariant of Grothendieck toposes which, informally, can be thought of as classifying internal symmetries of a topos. Algebraically the isotropy group generalizes the notion of centralizer, and is closely related to crossed modules. It was shown in [**?**] that localic toposes, and in particular toposes which can be identified as $Sh(\mathbb{B})$, have trivial isotropy groups. In the light of Freyd's result, it therefore makes sense to study the behaviour of the remaining two constructions.

In this work we describe the isotropy group $Z$ of the topos $\mathcal{B}(G)$ of continuous $G$-sets and $G$-equivariant maps in elementary terms. We also look at some properties of $Z$ and exhibit a subgroup of $Z$ which arises from the geometric morphism $\mathcal{B}(G^\delta) \to \mathcal{B}(G)$ induced by the continuous group homomorphisms $G^\delta \to G$, where $G^\delta$ is the topological group $G$ equipped with the discrete topology. We show that this subgroup is maximal in the case of infinite permutation groups.

Freyd,P., All topoi are localic, or why permutation models prevail, *Journal of Pure and Applied Algebra*, 46(1):49–58, 1987.

Funk,J., Hofstra,P., and Steinberg,B., Isotropy and crossed toposes, *Theory and Applications of Categories*, 26(24):660–709, 2012.

## Construction of categorical bundles from gerbes

(joint work with A. Lahiri, and A. N. Sengupta)

Saikat Chatterjee

A categorical principal bundle is a structure analogous to a classical principal bundle, where topological spaces are replaced with (topological) categories, Lie groups are replaced with Lie 2 groups and maps are replaced with functors; examples arise from geometric contexts involving bundles over path space groupoids. This work deals with local trivializations of such bundles. We show how a categorical principal bundle over a path space grouopid can be constructed from local data specified through nonabelian gerbes. We also give an interpretation of our construction in terms of extension of categorical groups and "quotient category".

## Higher Toda brackets and the Adams spectral sequence

(joint work with M. Frankland)

J. Daniel Christensen

I will review the construction of the Adams spectral sequence in a triangulated category equipped with a projective or injective class. Then, following Cohen and Shipley, I'll explain how to define higher Toda brackets in a triangulated category. The main result is a theorem which gives a relationship between these Toda brackets and the differentials in the Adams spectral sequence.

## Connections in Tangent Categories

(joint work with G. Cruttwell)

Robin Cockett

Tangent categories [1,2] provide a convenient setting for developing abstract differential geometry. They include, besides the classical settings for differential geometry, both Synthetic Differential Geometry (SDG), algebraic geometry, combinatorial, and Computer Science settings in which differentiation is used. A tangent category is a category with a functor which is axiomatized to behave like a tangent bundle. This allows one to define the analogue of the notion of a "vector bundle" (which we call a differential bundle [3]), and it is on these that one can define the general notion of a connection.

Connections play a key role in differential geometry: the purpose of this talk is to explore this notion at the abstract level of tangent categories and its relationship to sprays.

[1] Cockett, J.R.B. and Cruttwell, G.S.H. *Differential structure, tangent structure, and SDG.* Applied categorical structures. 2014.
[2] Rosický, J. Abstract tangent functors. *Diagrammes*, 12, Exp. No. 3, 1984.
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## A simplicial foundation for de Rham cohomology in a tangent category

(joint work with R. Lucyshyn-Wright)

Geoff Cruttwell

Tangent categories [1,5] provide an axiomatic framework for understanding various tangent bundles that occur in differential geometry, algebraic geometry, and elsewhere in mathematics. Previous work has shown that one can formulate and prove a variety of definitions and results from differential geometry in an arbitrary tangent category, including generalizations of vector fields and their Lie bracket [2], vector bundles [3], and connections.

In this talk we show how to define de Rham cohomology for a tangent category. In particular, we will consider a notion of "sector forms" in tangent categories (in the context of smooth manifolds, the definition is originally due to White [6]). We show that these sector forms have a rich structure: they form a symmmetric cosimpicial object in the sense of Grandis [4]. This appears to be a new result in differential geometry. When restricted to alternating sector forms, the resulting complex recovers the ordinary de Rham complex in the category of smooth manifolds.

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[2] J. R. B. Cockett and G. S. H. Cruttwell, The Jacobi identity for tangent categories, *Cah. Topol. Géom. Différ. Catég.*, **LVI** (4) (2015), 301–316.
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## Restriction monads

Darien DeWolf

Restriction monads are monads equipped with additional structure encoding the data of restriction categories in a more abstract setting. Specifically, restriction monads are defined so that restriction monads in **Span(Set)** are small restriction categories. This talk will introduce restriction monads, give examples, and will relate them both to double restriction categories and the internal definition of restriction categories.

## On open locales and inner products

J.M.Egger

Given an open locale $E$, the operation $[\alpha, \beta] = \exists_!(\alpha \wedge \beta)$ defines a sort of "inner product" on the underlying frame of $E$. In this talk, we explore fruitful analogies between the theory of inner product spaces and open locales, touching on the theory of uniform and metric locales, as well as modal logic and orthomodular lattices.

## 2-Crossed Modules of Hopf Algebras

(joint work with J.F. Martins)

Kadir Emir

In this work, we introduce the notion of a 2-crossed module of Hopf algebras, which generalize differential 2-crossed modules (Lie 3-algebras), 2-crossed modules of groups (3-groups) and also crossed modules of Hopf algebras.
Afterwards, we define the Moore complex of a simplicial cocommutative Hopf algebra by using Hopf kernels. These are defined in quite a different way in comparison to the kernels of morphisms of other algebraic structures: groups, Lie algebras, commutative algebras, etc. We prove that the category of simplicial cocommutative Hopf algebras with Moore complex of length two is equivalent to the category of 2-crossed modules of cocommutative Hopf algebras. This unifies previous similar results which hold in the categories of groups and of Lie algebras.

## Infinitary first-order categorical logic

Christian Espíndola

It has been known for decades that there is a classical relation between infinitary model theory and large cardinals. Natural questions about the completeness of infinitary logics with respect to set-valued models require the use of weakly/strongly compact cardinals, which stand out in the large cardinal hierarchy beyond inaccessibility. In this talk we will attempt to clarify this relationship in categorical terms, by considering a class of categories which have all the required properties for the interpretation of infinitary first-order logic. Previous work by Makkai in 1990 uses infinitary regular categories to describe the fragment of infinitary regular logics, allowing infinitary conjunction and infinitary existential quantification. We will introduce also infinitary disjunctions into the picture, so that the resulting logic, infinitary coherent, is expressive enough to support infinitary first-order theories. We present completeness theorems of infinitary coherent logic in terms of models in the corresponding infinitary coherent categories that we introduce, as well as with respect to set-valued models, generalizing the known completeness results for classical infinitary first-order logic. We present as well completeness theorems for infinitary first-order theories in terms of models in both sheaf and presheaf toposes, and we analyze to what extent the use of large cardinal axioms (more precisely, the condition that $\kappa$ be weakly compact) is necessary in each case. Some applications of the completeness results will also be presented.

[1]C. Espíndola, *Achieving completeness: from constructive set theory to large cardinals*, PhD thesis, 2016.
[2]M. Makkai, A theorem on Barr-exact categories, with an infinite generalization, *Annals of Pure and Applied Logic*, vol. 47 (1990), pp. 225–268.
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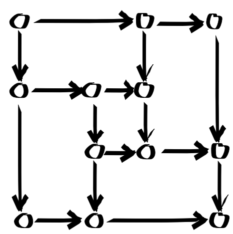
## Algebraic models of homotopy type theory

(joint work with C. Sattler)

Nicola Gambino

The distinction between ‘property’ and ‘structure’ arises in several parts of category theory, including categorical logic. In this talk, I will illustrate a further example of the difference between the satisfaction of a property and the presence of algebraic structure, which arises in the study of models of homotopy type theory in categories of simplicial sets and cubical sets. In particular, I will show how shifting the attention from standard weak factorisation systems (in the sense of Bousfield) to algebraic weak factorisation systems (in the sense of Grandis & Tholen, Garner) allows us to address some constructivity issues in the definition of the simplicial model of homotopy type theory. As a byproduct of this work, we obtain a new proof of the right properness of the Quillen model structure for Kan complexes.



## The centralizer of a topos

(joint work with P. Hofstra)

Jonathon Funk

An object of the centralizer $\mathcal{Z}(\mathbb{X})$ of a category $\mathbb{X}$ is a pair $(X, a)$, where $a$ is an automorphism of the identity functor on the slice category $\mathbb{X}/X$: we say that $a$ is a central automorphism of $X$. A morphism $f : (X, a) \to (Y, b)$ of $\mathcal{Z}(\mathbb{X})$ is a morphism $f : X \to Y$ of $\mathbb{X}$ such that the whiskering equation $b\Sigma_f = \Sigma_f a$ holds.

My explanations of the following assertions are based on isotropy theory for toposes [1].
(i) If $\mathbb{X} = \mathcal{E}$ is a Grothendieck topos, then so is $\mathcal{Z}(\mathcal{E})$. In fact, $\mathcal{Z}(\mathcal{E})$ is a connected, atomic quotient (in the sense of geometric morphisms) of the topos $\mathcal{E}/Z$ of crossed sheaves on $\mathcal{E}$, where $Z$ denotes the isotropy group of $\mathcal{E}$.
(ii) $\mathcal{Z}(\mathcal{E})$ is equivalent to the topos of unital crossed sheaves on $\mathcal{E}$: a crossed sheaf $t : X \to Z$ is unital if $\forall x \in X : xt(x) = x$.
(iii) The canonical braided tensor on crossed sheaves, which is balanced in the sense of Joyal and Street [2], pushes down to a closed symmetric monoidal structure on $\mathcal{Z}(\mathcal{E})$.

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## Categorical models of differential logic

(joint work with R. Cockett)

Jonathan Gallagher

It is a well known result due to Scott and Koymans that that models of the $\lambda$-calculus are exactly reflexive objects in a cartesian closed category [3,5]. The differential $\lambda$-calculus of Ehrhard and Regnier extends the $\lambda$-calculus with structure that allows differentials of functions [2]: the $\lambda$-calculus is to functions as the differential $\lambda$-calculus is to smooth functions.

Manzonetto investigated categorical models of the differential $\lambda$-calculus [4] using *linear* reflexive objects in a cartesian closed differential category, in which the section and retraction are both linear, but failed to provide an analog of the Scott-Koymans theorem. The difficulty is that cartesian differential categories are not closed to splitting arbitrary idempotents. Moreover, splitting the linear idempotents, whilst providing again a differential category, does not suffice: for example the idempotent that encodes pairs in the differential $\lambda$-calculus is not linear. However, one can split the *retractively linear* idempotents; these have the property that when split, the retraction is linear. This allows a full Scott-Koymans theorems to be developed for the differential $\lambda$-calculus.

From here, it is natural to consider type theory for the differential $\lambda$-calculus. We will argue that a natural setting for this extension is tangent categories [1].

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## 2-linear algebra and the Picard group of Picard categories

(joint work with N. Johnson and Angélica Osorno)

Nick Gurski

Linear algebra begins with the study of abelian groups, with one of the major drivers of the theory being the closed, symmetric monoidal structure given by the tensor product. One approach to higher categorical linear algebra is to replace abelian groups with Picard categories: symmetric monoidal categories in which both objects and morphisms are invertible. A basic question is then to ask if there is a closed, symmetric monoidal structure on the 2-category of Picard categories which categorifies the closed structure on abelian groups. A similar question has been studied in the wider context of all symmetric monoidal categories, and I will discuss what more can be said when restricting to Picard categories. Of particular interest is translating the categorified algebra back into traditional algebra, culminating in a proof that every invertible Picard category is equivalent to the unit, i.e., that the Picard group of Picard categories is trivial.

## A classifying topos for Penrose tilings

André Joyal

In his book on non-commutative geometry, Alain Connes observes that the "space" of Penrose tilings is "non-commutative" and he associates a $C^*$-algebra. We will show that the notion of (Penrose) tiling is geometric (in the topos theoretic sense). It follows that Penrose tilings have a classifying topos.

## Dagger category theory: monads and limits
(joint work with C. Heunen)

Martti Karvonen

A dagger category is a category equipped with a dagger: a contravariant involutive identity-on-objects endofunctor. Such categories are used to model quantum computing [3] and reversible computing [1], amongst others. The philosophy when working with dagger categories is that all structure in sight should cooperate with the dagger. This causes dagger category theory to differ in many ways from ordinary category theory. Standard theorems have dagger analogues once one figures out what "cooperation with the dagger" means for each concept, but often this is not just an application of formal 2-categorical machinery or a passage to (co)free dagger categories. We will discuss two instances.

First, as soon as a monad on a dagger category satisfies the Frobenius law, everything works as it should. Dagger adjunctions give rise to such monads. Conversely, such monads factor as dagger adjunctions in two canonical ways;

however, the Eilenberg-Moore category needs to be adapted to inherit the dagger [2].

Second, limits in dagger categories should be unique up to an unique unitary, that is, an isomorphism whose inverse is its dagger. We rework an initial attempt [4] to a more elegant and general theory. It works well when the diagram has a dagger; however, the formulation of limits using adjunctions needs to be adapted. We will discuss work on defining dagger limits of general diagrams.

Finally, we will discuss formally how dagger categories, while perhaps slightly evil, are not all that bad.

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## Categories of Filters as Fibered Completions

Toshiki Kataoka

It is well known that the poset $\mathfrak{F}L$ of filters on a (meet-)semilattice $L$ is a completion of $L$ in the sense that the functor $\mathfrak{F}$ is left adjoint to the forgetful functor $\bigwedge - \mathbf{SLat} \to \mathbf{SLat}$ where $\mathbf{SLat}$ denotes the category of semilattices and maps preserving finite meets and $\bigwedge - \mathbf{SLat}$ denotes the category of *complete* semilattices and maps preserving arbitrary meets. It was shown in [1] as an analogue that the category $\mathfrak{F}\mathcal{C}$ of filters on objects of a category $\mathcal{C}$ is a completion of $\mathcal{C}$. Precisely, it is characterized by adjoint functors between the category $\mathbf{Lex}$ of finitely complete categories and the category $\bigwedge^{\mathrm{filt}} - \mathbf{Lex}$ of finitely complete categories with filtered meets of subobjects.

We propose a category $\mathbf{LexFib}$, whose object is a fibered semilattice satisfying certain properties that are valid for subobject fibrations $\mathrm{SubFib}(\mathcal{C}) = \big(\mathrm{Sub}(\mathcal{C}) \to \mathcal{C}\big)$. We study adjoint functors $L \dashv \mathrm{SubFib} \dashv R\colon \mathbf{LexFib} \to \mathbf{Lex}$. The functor $R$ turns out to be a localization of categories, while $L$ is rather trivial. Following the above idea, we define the fibered completion as a left adjoint to a forgetful functor, and obtain

$$\mathbf{Lex} \rightleftarrows \mathbf{LexFib} \rightleftarrows \bigwedge^{\mathrm{filt}} - \mathbf{LexFib} \ .$$

The induced monad on $\mathbf{Lex}$ is shown to be equivalent to the monad induced by $\mathbf{Lex} \rightleftarrows \bigwedge^{\mathrm{filt}} - \mathbf{Lex}$ This result connects two "completions" $\mathfrak{F}$ of semilattices and of finitely complete categories, and gives an insight why the concrete definition of filter categories $\mathfrak{F}\mathcal{C}$ have to be complicated.

We conjecture similar adjunctions $\mathrm{SubFib} \dashv R\colon \mathbf{RegFib} \to \mathbf{Reg}$ and $\mathrm{SubFib} \dashv R\colon \mathbf{CohFib} \to \mathbf{Coh}$ for regular (resp. coherent) categories and regular (resp. coherent) fibrations. At least we have such adjunctions on full subcategories $\mathbf{RegFib}' \subseteq \mathbf{RegFib}$ and $\mathbf{CohFib}' \subseteq \mathbf{CohFib}$ containing fibered completions of subobject fibrations. From a logical perspective, we add a fibrational explanation to filter logics [1]: while a regular (resp. coherent) category with filtered meets is a categorical model of the regular (resp. coherent) filter logic, a regular (resp. coherent) fibration with fibered filtered meets is a fibrational model of it.

These constructions $\mathbf{RegFib}' \to \mathbf{Reg}$, $\mathbf{CohFib}' \to \mathbf{Coh}$ are so new that they are (in general) different from the standard constructions $\mathrm{FRel}\colon \mathbf{RegFib} \to \mathbf{Reg}$, $\mathrm{FRel}\colon \mathbf{CohFib} \to \mathbf{Coh}$, where a morphism in $\mathrm{FRel}(p)$ is a functional relation.

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## Motivic derived algebraic geometry and loop stacks of the affine stacks of periodic motivic $\mathbb{E}_\infty$-rings

Yuki Kato

The theory of motivic derived algebraic geometry is an enhancement of derived algebraic geometry for the direction of $\mathbb{A}^1$-homotopy theory introduced by Morel and Voevodsky [9]. In the framework of the theory of motivic derived algebraic geometry, we define motivic versions of $\infty$-categories, $\infty$-topoi, classifying $\infty$-topoi, spectral schemes and spectral Deligne–Mumford stacks which are defined by Joyal [3,4] and Lurie [6,8]. By using the theory of motivic derived algebraic geometry, we study the relation of the affine (motivic) stacks of some interesting periodic motivic $\mathbb{E}_\infty$-rings (for example, the periodic sphere, $K$-theory spectrum and the periodic algebraic cobordism) and the loop stacks of these affine stacks.

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[7] Lurie, J., $(\infty, 2)$-*categories and the Goodwillie calculus I*, preprint, 2009, available at www.math.harvard.edu/lurie
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## Computing Isotropy in Grothendieck Toposes

Sakif Khan

In the paper [1], the authors announce the discovery of an invariant for Grothendieck toposes which they call the isotropy group of a topos. Roughly speaking, the isotropy group of a topos carries algebraic data in a way reminiscent of how the subobject classifier carries spatial data. Much as we like to compute invariants of spaces in algebraic topology, we would like to have tools to calculate invariants of toposes in category theory. More precisely, we wish to be in possession of theorems which tell us how to go about computing (higher) isotropy groups of various toposes. As it turns out, computation of isotropy groups in toposes can often be reduced to questions at the level of small categories and it is therefore interesting to try and see how isotropy behaves with respect to standard constructions on categories. We aim to provide a summary of progress made towards this goal, including results on various commutation properties of higher isotropy quotients with colimits and the way isotropy quotients interact with categories collaged together via corepresentable profunctors. The latter should be thought of as an analogy for the Seifert-van Kampen theorem, which allows computation of fundamental groups of spaces in terms of fundamental groups of smaller subspaces.

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## A categorial approach to the maximum theorem

Seerp Roald Koudenburg

Given a relation $J \subseteq A \times B$ between topological spaces and a continuous map $d \colon B \to [0, \infty]$, consider the function $r \colon A \to [0, \infty]$ given by the suprema

$$rx = \sup d\{y \in B \mid (x, y) \in J\}.$$

Berge's maximum theorem, which is used in mathematical economics, gives conditions on the relation $J$ ensuring the continuity of $r$.

Using the extension of the ultrafilter monad to the double category of relations we will sketch a categorical proof of the maximum theorem. Replacing the ultrafilter monad by related monads then allows us to obtain variations of the maximum theorem, such as for closure spaces, approach spaces and metric closure spaces.

## Quantale-valued Approach Spaces via Closure and Convergence

(joint work with W. Tholen)

Hongliang Lai

For a quantale $(\mathsf{V}, \otimes, \mathsf{k})$ we introduce $\mathsf{V}$-approach spaces via $\mathsf{V}$-valued point-set-distance functions and, when $\mathsf{V}$ is completely distributive, characterize them in terms of both, so-called closure towers and ultrafilter convergence relations based on the notions of lax distributive laws and lax algebras [3,6]. When $\mathsf{V}$ is the two-element chain $\mathsf{2}$, the extended real half-line $[0, \infty]$, or the quantale $\boldsymbol{\Delta}$ of distance distribution functions, the general setting produces known and new results on topological spaces [1], approach spaces [5], and the only recently considered probabilistic approach spaces [2,4], as well as on their functorial interactions with each other.

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## Introduction to Cartesian Integral Categories

(joint work with R. Cockett, R. Blute,and C. Bauer)

JS Lemay

The two fundamental theorems of calculus tie together key concepts of calculus and analysis: differentiation and integration. One would hope that category theory would be able formalize differentiation and integration so that these fundamental theorems hold. Blute, Cockett and Seely introduced the notions of both Cartesian differential categories [1,2] and monoidal differential categories [3], which formalize differentiation. Differentiation in category theory has now been extensively studied, however, integration has not received so much attention.

In this talk, we will introduce Cartesian integral categories. The axioms involve integrating maps which are linear in the second argument [2], which must satisfy the integral formulas for polynomials and Fubini's theorem. When the integral structure is compatible with the Cartesian differential structure, we obtain a setting in which both of the fundamental theorems of calculus hold. We will also describe how anti-derivation give rise to an integral structure. To give intuition for this talk, we will present a term logic for integration (analogous to that in [1] for differentiation).

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# Biadjoint Triangles

Fernando Lucatelli Nunes

There are several adjoint triangle theorems in the literature [7]. In [6], we give a 2-dimensional analogue of the adjoint triangle theorem of Dubuc [1]. Therein, we also study the counit and unit of the obtained biadjunction and give consequences of the main theorem, such as the pseudomonadicity characterization [3] and the coherence theorem due to Lack on strict replacements of pseudoalgebras of a 2-monad [2].

In this talk, we will show a proof of the adjoint triangle theorem of Dubuc and show how it works for the 2-dimensional case. To do so, we will prove the biadjoint triangle theorem as a consequence of a basic theorem on (pseudo)premonadic (pseudo)functors and Descent [4,5,8], making comments on further work [6]. If time permits, we will talk about applications, such as the general coherence theorem of Lack mentioned above.

This work is part of my PhD studies under supervision of Maria Manuel Clementino at University of Coimbra.

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[4] F. Lucatelli Nunes. On Biadjoint Triangles *Theory Appl. Categ.* 31 (2016), no. 9, pp 217-256.

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[6] F. Lucatelli Nunes. A coherent approach to biadjoint triangles (in preparation).

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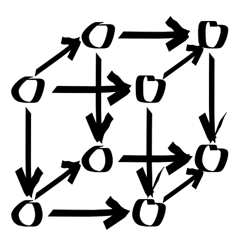

# Enriched algebraic theories,monads, and commutants in the foundations of categorical distribution theory

Rory Lucyshyn-Wright

In this talk we survey the speaker's recent work on $\mathcal{V}$-enriched $\mathcal{J}$-algebraic theories for a *system of arities* $\mathcal{J} \hookrightarrow \mathcal{V}$ in a symmetric monoidal closed category $\mathcal{V}$ [1, 2], with attention to the notion of *commutant* for $\mathcal{J}$-theories [2, 3] and its applications in categorical functional analysis [4]. For suitable choices of $\mathcal{J}$, such $\mathcal{J}$-theories include Lawvere's algebraic theories, Linton's equational theories, the enriched algebraic theories of Borceux and Day, the enriched Lawvere theories of Power, and the $\mathcal{V}$-theories of Dubuc, which correspond to arbitrary $\mathcal{V}$-monads on $\mathcal{V}$. We prove theorems on the existence and characterization of the $\mathcal{V}$-categories of $\mathcal{T}$-algebras for $\mathcal{J}$-theories $\mathcal{T}$. We show that several results are enabled by the modest requirement that $\mathcal{J}$ be *eleutheric*, equivalently that the inclusion $\mathcal{J} \hookrightarrow \mathcal{V}$ present $\mathcal{V}$ as a free cocompletion of $\mathcal{J}$ with respect to certain weights. We show that $\mathcal{J}$-theories for an eleutheric system are equivalently described as *$\mathcal{J}$-ary monads*, certain enriched monads on $\mathcal{V}$. We discuss the notion of *commutant* of a morphism of $\mathcal{J}$-theories, generalizing Wraith's notion of commutant for Linton's equational theories. We obtain notions of *$\mathcal{J}$-ary commutant* and *absolute commutant* for $\mathcal{J}$-ary monads, and we show that for finitary monads on Set the resulting notions of finitary commutant and absolute commutant coincide. We discuss the speaker's recent results on commutants of the Lawvere theories of $R$-affine spaces for a rig $R$ and $R$-convex spaces for a preordered ring $R$ [3]. We review how the notion of *canonical distribution monad* [4] is defined in terms of commutants for $\mathcal{J}$-theories and captures several examples of monads of measures and distributions.

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## Equivalences of (intensional) type theories and their models

(joint work with K. Kapulkin)

Peter LeFanu Lumsdaine

In many logical systems — predicate logic and its various fragments, for instance, or Martin-Löf's extensional dependent type theory — the connectives and their interpretations enjoy 1-categorical universal properties, determining them up to canonical isomorphism. This allows for a good theory of natural isomorphisms between parallel interpretations of such systems, and consequently the use of equivalences of (1-)categories as a notion of equivalence between theories or models of such systems.

In intensional dependent type theory, however, this breaks down. Connectives are determined only up to weaker notions of equivalence, so natural isomorphisms and equivalences of categories are too restrictive as notions of equivalence between models — much as in other settings, such as Quillen model categories, where 1-categorical structures are naturally seen as presentations of $(\infty, 1)$-categories. We present an alternative approach, based essentially on an interpretation of the logic in *Reedy span-equivalences.* The resulting notions of equivalences between models suffice for applications of several different flavours:

- a higher-categorical analysis of $(\infty, 1)$-categories of "intensional type theories";
- inter-interpretability and conservativity results between different theories;
- strong invariance/uniqueness statements for interpretations of the logical connectives.

## Partial Fermat theories and differential restriction theories

(joint work with R. Cockett and J. Gallagher)

Ben MacAdam

Cartesian differential categories [1] and Fermat theories [2,5] are both axiomatizations of smooth functions in a Cartesian space. It's unsurprising that the differential structure of a Fermat theory gives a Cartesian differential theory; in fact, Cartesian differential theories are strictly more general than Fermat theories. However, while Cartesian differential categories have a natural extension to partial functions, i.e. differential restriction categories [4], no such notion has been developed for Fermat theories. In this talk we will begin by defining partial Fermat theories, both with and without joins, and explain some examples, particularly locally analytic partial functions on a $p$-adic vector space [6]. We will show how the differential structure of a partial Fermat theory yields a differential restriction category, and how we may apply these constructions to build manifolds from a partial Fermat theory with joins [3].

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## Tertiary homotopy operations in a 2-category

(joint work with N. Oda)

Howard J. Marcum

Secondary and higher order homotopy operations (Toda brackets) were introduced by Toda in order to construct elements of the homotopy groups of spheres as part of his "composition method" for computing these groups. As early as the 9-stem a tertiary operation (quaternary Toda bracket) was needed to describe a generator. Unfortunately a coherence condition and considerable detail obscure the definiton of this operation.

Beginning in the mid-1990's work of Hardie, Kamps, Marcum, Oda ([1 - 5]) and others have developed classical (and newer) secondary homotopy operations in the setting of a 2-category with zeros. If the given 2-category $\mathcal{C}$ has additional structure — such as a suspension 2-functor $\Sigma\colon \mathcal{C} \to \mathcal{C}$ with a related double mapping cylinder 2-functor (as we assume and develop) — then $\mathcal{C}$ admits a theory of extensions and coextensions, and accordingly tertiary operations can be defined. We define several such new operations and focus particular attention on a new tertiary operation called the *box quaternary operation*. New insights on the special case of the classical quaternary Toda bracket are obtained.

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## Categorical Foundations of Big Data Analytics

Yoshihiro Maruyama

Among a tremendous number of methods proposed so far in Artificial Intelligence, the so-called Kernel Method is known as providing one of the most successful technologies in practice use, especailly in trendy Big Data Analytics. Underpinning the Kernel Method is a certain relationship between so-called kernel functions (conceptually representing similarity functions) and reproducing kernel Hilbert spaces. In this work we aim at elucidating the link between kernel functions and reproducing kernel Hilbert spaces by means of Categorical Duality Theory. Regarding kernel functions as Chu spaces, we show that there are indeed dualities between categories of kernel Chu spaces and categories of reproducing kernel Hilbert spaces. Although our primary emphasis is put upon purely conceptual facets of Machine Learning, we nevertheless speculate about potential practical applications of coceptual insights obtained via the Duality Theory of Machine Leraning.

## The construction of $\pi_0$ in the context of Axiomatic Cohesion

M. Menni

Let $p^* \dashv p_* : \mathcal{E} \to \mathcal{S}$ with $\mathcal{S}$ a topos and $\mathcal{E}$ a cartesian closed regular category. Following a suggestion by Lawvere we construct a functor $\pi_0 : \mathcal{E} \to \mathcal{E}$

which may be thought of as assigning, to each object $X$ in $\mathcal{E}$, the object of 'connected components of $X$'. To justify this claim we apply the construction to prove a characterization of the local and hyperconnected geometric morphisms $p : \mathcal{E} \to \mathcal{S}$ that are pre-cohesive. We also illustrate the construction in the case that $\mathcal{S} = \mathbf{Set}$ and $\mathcal{E}$ is the category of compactly generated Hausdorff spaces.

## Turing Categories and Realizability

(joint work with R. Cockett)

Chad Nester

Associated with every partial combinatory algebra is a realizability topos, which may be constructed either through tripos theory, or through the related category of assemblies. These constructions rely on the logic of the category of sets and partial functions. Turing categories give a more general presentation of partial combinatory algebras, implying that the realizability topos must be constructed more abstractly. To accomodate this, we show sets and partial functions can be replaced by any discrete cartesian closed restriction category. In this setting we construct the category of assemblies, the realizability tripos, and thus the realizability topos.

Along the way we show that every discrete cartesian closed restriction category gives rise to a tripos. Thus, from an arbitrary discrete cartesian closed restriction category we show how to construct a related (partial) topos.

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## Categories of Natural Models of Type Theory

(joint work with S. Awodey)

Clive Newstead

Natural models of type theory (see Awodey, arXiv:1406.3219v2) provide an algebraic setting for the interpretation of dependent type theory, based on the notion of a representable natural transformation of presheaves. First, we define homomorphisms of natural models of type theory with a basic type and context extension, by analogy to morphisms of sheaves, and prove that the syntax is initial in this category. We then extend our construction to allow for dependent sums and products. Time permitting, we prove that, in this case, the polynomial functor associated with a given natural model gives rise to a polynomial monad. This is joint work with Steve Awodey.

## Coexponentiability and Projectivity: Rigs, Rings, and Quantales

Susan Niefield

Let $R$ be a rig, i.e., a *ring without negatives.* Then $R$ is a commutative monoid in the symmetric monoidal category of commutative monoids, and $\otimes_R$ is the coproduct in the category $\mathrm{Rig}\backslash R$ of rigs under $R$. In this talk, we show that a homomorphism $R \to C$ is coexponentiable, i.e., the functor $- \otimes_R C$ has a left adjoint, if and only if $C$ is finitely generated and projective as an $R$-module. To do so, we prove a general theorem for symmetric monoidal closed categories $\mathcal{V}$. In particular, we show that a commutative monoid $C$ is coexponentiable in $\mathrm{CMon}(\mathcal{V})$ if and only if the functor $- \otimes C \colon \mathcal{V} \to \mathcal{V}$ has a left adjoint, and the latter is characterized by a condition that is equivalent to $C$ being finitely generated and projective in the case where $\mathcal{V}$ is the category of modules over a rig $R$ so that $\mathrm{CMon}(\mathcal{V}) \cong \mathrm{Rig}\backslash R$. The general theorem also gives previously known analogous characterizations of coexponentiable morphisms of commutative rings and quantales.

## Strictification tensor product of 2-categories

Branko Nikolić

Monads in a 2-category $\mathcal{E}$ correspond to lax functors from the terminal 2-category $\mathbf{1}$ into $\mathcal{E}$, which are also in bijection with strict 2-functors from the free monad [1] 2-category FM into $\mathcal{E}$ . Moreover, with $\mathrm{Lax}(-,-)$ denoting a 2-category of lax functors, lax natural transformations and modifications, and $[-,-]_{lnt}$ being its full sub-2-category consisting only of strict 2-functors, there are isomorphisms $\mathrm{Mnd}(\mathcal{E}) \cong \mathrm{Lax}(\mathbf{1},\mathcal{E}) \cong [\mathrm{FM},\mathcal{E}]_{lnt}$.

Similarly, distributive laws can be viewed as monads in $\mathrm{Mnd}(\mathcal{E})$, and so as the objects of a 2-category isomorphic to $\mathrm{Lax}(\mathbf{1},\mathrm{Lax}(\mathbf{1},\mathcal{E}))$. The latter is isomorphic to the 2-category $[\mathrm{FDL},\mathcal{E}]_{lnt}$ where FDL is the free distributive law whose dual construction was presented in [2].

Given two 2-categories $\mathcal{C}$ and $\mathcal{D}$ we describe a 2-category $\mathcal{C} \boxtimes \mathcal{D}$ satisfying

$$\mathrm{Lax}(\mathcal{C},\mathrm{Lax}(\mathcal{D},\mathcal{E})) \cong [\mathcal{C} \boxtimes \mathcal{D},\mathcal{E}]_{lnt}$$

therefore generalizing the case $\mathrm{FDL} = \mathbf{1} \boxtimes \mathbf{1}$. We present two isomorphic constructions, one using computads [3], and one using the simplex category of intervals [4]. We also modify the construction to describe various duals and provide examples.

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## Span equivalence between weak $n$-categories

Yuya Nishimura and Hiroyuki Miyoshi

What is an appropriate notion of "equivalence" between weak $n$-categories defined by globular operads? In this talk, we propose an answer for this question.

T. Cottrell defined a notion of equivalence for his coherence theorem in [1]. As pointed out in that paper, however, this notion is not an equivalence relation. Instead, we introduce the notion of *span equivalence* which overcomes this and still allows essentially the same coherence theorem.

Span equivalence is indeed an equivalence relation. For transitivity, we show some properties are stable under pullback, and the pullback of span equivalences turns out to be a span equivalence. Then we can see the transitivity easily.

Furthermore, in **Cat**, we can construct a span equivalnce structure from a pair of two (adjointly) equivalent categories through a construction here called *equivalence fusion*, and the converse is staightforward. In this sense, span equivalences are an extension of equivalences in **Cat**.

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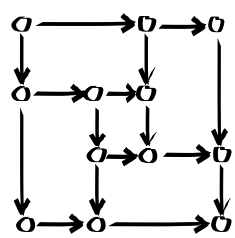

## Smoothness in Differential Categories

(joint work with R. Blute)

Keith O'Neill

Various notions of smoothness exist in both Algebraic Geometry and Noncommutative Geometry. In the present work the structure of differential categories is utilized in order to encapsulate these various notions in a common framework; consequently, a notion of smoothness is proposed for models of differential linear logic.

The examination of smoothness here is facilitated primarily by ideas from André-Quillen homology, which have been adapted to differential categories. With these it can be shown that a smooth T-algebra in a codifferential category satisfies a condition, which is a generalization of the Höchschild-Kostant-Rosenberg theorem.

## Isotropic Intercategories

Robert Paré

Intercategories are a special kind of lax triple category which might be viewed as "duoidal categories with several objects". Certain intercategories of interest, including Gray categories and locally cubical bicategories, admit a quintet construction. The resulting intercategories, which we call *isotropic*, are characterized.

## Mapping Spaces for Orbispaces

(joint work with L.Scull)

Dorette Pronk

In this talk we introduce and study mapping spaces for orbispaces. We define orbispaces using proper etale topological groupoids, which we call orbigroupoids. Then the bicategory of orbispaces is the bicategory of fractions of orbigroupoids with respect to essential equivalences. The first problem with defining topological mapping groupoids is the fact that the canonical mapping groupoids are not small. In order to get a small mapping space, we develop some general results about bicategories of fractions: we will weaken the conditions on a class of maps needed to give rise to a bicategory of fractions, and show when inverting a subclass of maps gives an equivalent bicategory of fractions. This will enable us to define a topological groupoid OMap(G,H), encoding generalized maps in the bicategory of fractions from G to H, and the 2-cells between them. We will give a concrete description of this groupoid and study some of its properties.

We will then consider the case when the groupoid G is orbit-compact, i.e., when its underlying quotient space is compact. In this case we can further reduce the class of arrows to be inverted, which significantly simplifies the space of objects in the mapping groupoid OMap(G,H) and allows us to prove that this groupoid is etale and proper. Thus, we can produce a mapping object which is again an orbispace. This is a special case of a more general result which I may present if time permits: each mapping category in a bicategory of fractions C(W) with small hom-categories can be described as a pseudo colimit of hom-categories in the original bicategory C.

## Higher order chain rules for abelian functor calculus

(joint work with K.Bauer, B.Johnson, C.Osborne, and A.Tebbe)

Emily Riehl

Homological algebra studies functors between abelian categories that are typically required to be *additive*, preserving finite direct sums. Abelian *functor calculus* (developed by Johnson–McCarthy after Goodwillie) comes into play when this condition fails. To an arbitrary functor $F\colon \mathsf{A} \to \mathsf{B}$ between abelian categories, there is a "Taylor tower" of degree $n$ polynomial approximations $P_nF\colon \mathsf{A} \to \mathsf{ChB}$ that are universal up to quasi-isomorphism. In analogy with ordinary calculus, the degree 1 homogeneous approximations can be shown to satisfy the chain rule, with composition defined in the Kleisli category for a suitably defined chain complex monad.

In this project, we study iterates of the first-order directional derivative. In particular, we define a functor calculus analog of a new higher-order directional derivative introduced by Huang–Marcantognini–Young and prove that it satisfies a higher-order chain rule with an appealingly simple form. The reason that the complicated coefficients of the famous Faà di Bruno formula for higher-order chain rules do not appear is because these coefficients precisely govern the relationship between these new higher-order directional deriatives and a more straightforward iterated directional derivative studied by Johnson–McCarthy. The various lemmas required to prove these results assemble into a proof that the Kleisli category for the chain complex monad is a cartesian differential category in a suitably homotopical sense.

## Fibred Signal Representation

(joint work with S.D.Joshi)

Salil Samant

Signal representation is the most fundamental aspect of any signal processing application. Classically, a signal is viewed as an entity varying naturally in time or space. All classical approaches for signal representation model it as an element of a linear function space frequently with a group action on it, mainly utilizing the techniques from functional and harmonic analysis developed in the last century; refer [1] and references therein. In the last decade many new ways of representing signals are being considered to exploit some additional structure such as sparsity, topology and graph inherent in certain class of signals.

Consideration of additional algebraic structures necessitates and motivates one to consider an appropriate general structural mathematical framework. This would facilitate the unification of new schemes with the classical ones and precisely explain the gains achieved in terms of both compression and information analysis.

We propose a unifying framework for signal representation using the general formalism of fibred category theory [2] and introduce a notion of fibred signal representation, stressing the crucial base perspective. In this viewpoint the signal, instead of being viewed as a single vector in some single linear space, is modeled as residing in multiple linear spaces that form a total space fibred on a suitable base category comprising of objects corresponding to the true generators of signal whereas the morphisms capture natural relationship between the generators. Further by utilizing the cartesian lifts of the base morphisms to differentially represent the sub-signals, we observe the power of Grothendieck's relative point of view in a new applied context. In essence, the proposed framework shifts emphasis from the usual Banach and Hilbert spaces common in classical signal representation to general fibred spaces such as presheaves, sheaves and bundles.

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## A Generalized Globular Approach to (Parts Of) Higher Topos Theory

Lucius Schoenbaum

A category may be defined as a structure $(C, \cdot, s, t)$, consisting of a set $C$ equipped with a partial binary operation $(\cdot) : C \times C \to C$ and mappings $s, t : C \to C$, subject only to the conditions $(ab)c = a(bc)$; $s(ab) = sb$; $t(ab) = ta$; $ab$ defined if and only if $tb = sa$; $ta = sa = a$ implies ($ba$ defined implies $ba = b$) and ($ab$ defined implies $ab = b$); and finally, $tta = sta = ta$ and $tsa = ssa = sa$. The last axiom is extraneous, that is, it never arises in proofs. Dropping this axiom corresponds to raising the dimension (there is an adjunction between a subcategory in the newly defined setting and the category of globular sets) or (more radically) dropping the presence of dimension (or graded structure) altogether, leading to potentially extreme pathological cases (e.g., generalized "categories" with no objects). In spite of this rather vast new generality, however, the proven framework of category theory continues to function as it normally does. Not only do the basic mechanisms of category theory continue to function, but remarkably, one also still has (after some work) all the basic mechanisms of (Grothendieck) topos theory. As one might expect, some constructions are not difficult to generalize, however, others involve some nontrivial and perhaps new ideas, for example, in the case of the Yoneda embedding and associated density theorem, as well as Lawvere's comma category, the basic ingredient in the Grothendieck construction. The understanding of such generalized toposes, or what might be regarded as topos theory in a generalized globular setting, is far from complete, but in the author's opinion, the results obtained thus far warrant further investigation. A second avenue of research is the study of relationships to higher category theory. The author has recently undertaken the study of a particular mapping from higher categories into the space of such generalized categories with monoidal structures. The goal is to reduce via this mapping the complexity of higher coherence laws (to a degree that is still being ascertained via study of bicategories, tricategories, and $n$-category theory for small $n$). There is no lack of connections between this approach and contributions by other mathematicians, for example, those of Batanin and Leinster on globular operads and multicategories, Cockett and Lack on restriction categories, and Street, Walters, and others on bitoposes and higher topos theory. If time allows, these connections will be discussed along with a short report of findings.

## A New Notion of Orbifold Atlas

(joint work with D. Pronk)

Alanod Sibih

Orbifolds were introduced by Satake [2,3] in terms of an underlying space with an atlas of charts consisting of an open subset of Euclidean space with an action of a finite group. The representation of orbifolds by topological groupoids introduced by Moerdijk and Pronk [1] has greatly helped in developing their study: orbifolds are then proper effective etale groupoids. Motivated by examples from physics we have started considering orbifolds with non-effective group actions. This has led us to consider orbifolds as proper etale groupoids.

In this talk, we will introduce a new notion of non-effective orbifold atlas which corresponds closely to the groupoid representation of orbifolds. This definition uses the language of double categories, and in particular the double category $\mathbb{B}$imod of finite groups with group homomorphisms as vertical morphisms and bimodules as horizontal morphisms. We will describe how this new definition of orbifold atlas generalizes Satake's definition. As an application we will introduce a notion of atlas refinement and atlas equivalence which corresponds to Morita equivalence of groupoids. This will also lead us toward the right notion of maps between atlases.

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## String diagrams for traced and compact categories are oriented 1-cobordsisms

(joint work with P. Schultz and D. Rupel)

David I. Spivak

We will present an alternate conception of string diagrams as labeled 1-dimensional oriented cobordisms, the operad of which we denote by $\mathbf{Cob}_{/\mathcal{O}}$, where $\mathcal{O}$ is the set of string labels. The axioms of traced (symmetric monoidal) categories are fully encoded by $\mathbf{Cob}_{/\mathcal{O}}$ in the sense that there is an equivalence between $\mathbf{Cob}_{/\mathcal{O}}$-algebras, for varying $\mathcal{O}$, and traced categories with varying object set. The same holds for compact (closed) categories, the difference being in terms of variance in $\mathcal{O}$. Time permitting, we will give a characterization of the 2-category of traced categories solely in terms of those of monoidal and compact categories, without any reference to the usual structures or axioms of traced categories.

## Cherchez la Cogèbre

(joint work with R. Garner)

Ross Street

Behind many a good algebra there is a coalgebra. This is true of the pointwise algebra structure on the k-linear space $A^{\mathbb{N}}$ of sequences in a k-algebra $A$. It is true more interestingly of the $\lambda$-Hurwitz product [2] on $A^{\mathbb{N}}$. The way in which 2-dimensional pointed k-coalgebras $D$ and $C(\lambda)$ determine the pointwise and $\lambda$-Hurwitz products will be explained along with the relevance of a canonical pointed k-coalgebra morphism $D \to C(\lambda)$.

A categorical version of the last paragraph will be presented whereby $A^{\mathbb{N}}$ is replaced by the category $\mathcal{A}^{\mathfrak{S}}$ of Joyal species [3] in a "$\mathcal{V}$-algebra" $\mathcal{A}$; see [5]. This will be linked with monoidality of the equivalences of Dold-Kan type in the sense of [4].

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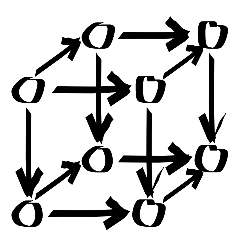

## Fractional Euler characteristics and why should we care?

Catharina Stroppel

In this talk I will explain what it means to categorify natural numbers, polynomials and rational numbers and why representation theorists and topologists might care. A leading example will be knot invariants or manifold invariants.

## Syntax and Sematics of Monad-Quantaloid Enrichment

(joint work with H. Lai)

Walter Tholen

For a **Set**-monad $T$ satisfying the Beck-Chevalley condition, there is a unique way of extending $T$ laxly and "flatly" from maps to relations, as first used by Barr in the case of the ultrafilter monad and then greatly generalized by Hofmann to the context of $\mathcal{V}$-valued relations, for a commutative quantale $\mathcal{V}$. A large part of the study of monad-quantale enriched categories in *Monoidal Topology* relies on this construction.

In this talk we replace the quantale $\mathcal{V}$ by a small quantaloid $\mathcal{Q}$ and consider the needed ingredients for a three-step procedure of extending a monad $T$ on $\mathbf{Set}/\mathrm{ob}\mathcal{Q}$, first to $\mathcal{Q}$-**Rel**, then to $\mathcal{Q}$-**Cat**, and finally to $\mathcal{Q}$-**Dist**, the category of small $\mathcal{Q}$-categories and their distributors (= bimodules). In fact, for the last step, exploiting a technique previously used to extend the Hausdorff monad, we give a criterion for any 2-monad $T$ of $\mathcal{Q}$-**Cat** to admit a (uniquely determined) lax flat extension to $\mathcal{Q}$-**Dist**. We discuss the bijective correspondence between lax extensions of $T$ and lax distributive laws of $T$ over the $\mathcal{Q}$-presheaf monad, as well as properties and examples of $(T, \mathcal{Q})$-enriched categories.

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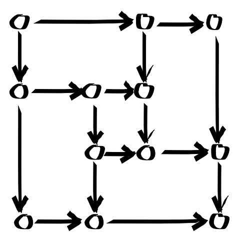
## Homotopy Model Theory

Dimitris Tsementzis

Set-theoretic foundations take sets to be their basic objects, but the Univalent Foundations of mathematics take homotopy types as their basic objects. Thus, a model theory in the framework of Univalent Foundations requires a logic that allows us to define structures on homotopy (n-)types, similar to how first-order logic can define structures on sets. We will define such an "n-level" logic for finite n. The syntax will be categorical in nature and is based on a generalization of Makkai's FOLDS, obtained by an operation that allows us to add equality sorts to FOLDS-signatures through an operation of globular completion". We then describe a homotopy-theoretic semantics for this logic and sketch relevant soundness and completeness results. Finally, as an application, we prove that univalent categories in the sense of the HoTT book are axiomatizable in 1-logic.



## Sketches in Higher Category Theories and the Homotopy Hypothesis

Rémy Tuyéras

A model structure on a category consists of three classes of maps, called the cofibrations, the weak equivalences, and the fibrations; in fact any two of these classes determine the third. Very often, these are determined by two sets, I and J, of cofibrations, called the generating cofibrations and the generating trivial cofibrations; the model structure is then said to be cofibrantly generated. Given any two sets I and J of morphisms in a category, it therefore makes sense to ask whether they generate a model structure. My first goal is to answer this question, and to characterize the resulting notion of weak equivalence.

I will do this by first characterizing when I and J generate something more general than a model structure, called a spinal structure; I will then characterize when this spinal structure is in fact a model structure. As an application, I shall construct a spinal structure on Grothendieck's category of $\infty$-groupoids, whose form is very close to the classical spinal (model) structure on topological spaces. Finally, I shall explain how these techniques might provide a proof of the Homotopy Hypothesis for (Grothendieck) $\infty$-groupoids in the near future.

## On The Yoga of (Un)Straightening

(joint work with E. Riehl)

Dominic Verity

The (un)straightening construction is one of the key technical devices introduced in Lurie's analysis of higher topos theory [1]. In essence, it provides an equivalence between the $\infty$-category of Cartesian fibrations (with small fibres) over a quasi-category $B$ and the $\infty$-category of functors from $B^{\mathrm{op}}$ into the $\infty$-category of all (small) quasi-categories $\mathcal{Q}$.

Whereas this result is often framed as a Quillen equivalence between suitable model categories, we prefer to render it as an equivalence of quasi-categories:

$$N_{\mathrm{hc}}(\mathrm{qCart}/B) \simeq \mathrm{fun}(B^{\mathrm{op}}, \mathcal{Q})$$

In this talk we will consider an elementary proof of this result, which proceeds by constructing an explicit functor from right to left and then applying a Beck monadicity argument to show that it is an equivalence. One novelty of this proof is that it relies in no way on any model category theory, since it is built only upon the elementary homotopy theoretic foundations developed in [2] and its companion papers.

Our interest in the techniques of this talk extends well beyond the pleasure of re-proving an important foundational result. At one level this rendition immediately rounds out our 2-categorical development of $(\infty, 1)$-category theory, by showing that the 2-category of quasi-categories (almost) admits a 2-topos structure, in the sense of Weber [5], and thus that it admits a natural family of Yoneda structures satisfying the Street and Walters axioms [3]. At another, generalisations of these techniques may also be applied in the Street and Verity *complicial sets* model of $(\infty, \infty)$-category theory [5], which allows us to contemplate the development of the category theory of such unrestricted weak higher categories.

[1] J. Lurie. *Higher Topos Theory*, volume 170 of *Annals of Mathematical Studies*. Princeton University Press, Princeton, New Jersey, 2009.
[2] E. Riehl and D. Verity. Homotopy coherent adjunctions and the formal theory of monads. *Adv. Math.*, 286:802–888, 2016.
[3] R. H. Street and R. Walters. Yoneda structures on 2-categories. *Journal of Algebra*, 50:350–379, 1978.
[4] D. Verity. Weak complicial sets I, basic homotopy theory. *Advances in Mathematics*, 219:1081–1149, September 2008.
[5] M. Weber. Yoneda structures from 2-toposes. *Applied Categorical Structures*, 15(3):259–323, 2007.

## Semistrict 4-categories and strong adjunctions

(joint work with K. Bar)

Jamie Vicary

We propose a new definition of semistrict 4-category, and present a formalization of it using the online proof assistant *Globular* [1]. As evidence for the correctness of our definition, we prove that every weak adjunction of 1-morphisms in a semistrict 4-category can be promoted to a strong adjunction satisfying the butterfly equations (formalized proof available at `http://globular.science/1605.002`). Our definition of semistrict 4-category has some connection to that proposed by Crans [2], but is distinct from it in some essential ways. We also describe progress towards a definition of semistrict $\infty$-category which is amenable to computer formalization.

[1] Krzysztof Bar, Aleks Kissinger and Jamie Vicary (2015). The proof assistant *Globular*, available at `http://globular.science`.
[2] Sjoerd Crans (1998). *On braidings, syllepses and symmetries.* Unpublished note.

# On the classification of cobordisms with strings

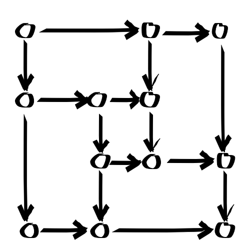
Jun Yoshida

In the paper [2], Joyal and Street developed a geometrical method, called the string calculus, of computations in some sorts of monoidal categories, which is widely used especially in the representation theory. Roughly, it is a calculus on "pictures" on the plane. On the other hand, we have another kind of calculus of pictures, topological field theories (TFT). According to Baez and Dolan's Cobordism Hypothesis ([1], see also [3]), a TFT captures a duality in a monoidal category.

Notice those notions above share a common ground. My work involves an attempt to unify them, namely TFT with singularities. Precisely, we consider cobordisms with submanifolds embedded. We are interested in the classification of cobordisms of such type. For this, two categories are introduced. The first one provides a "relative" notion for manifolds. Objects are called arrangements of manifolds, which are manifolds equipped with submanifolds whose intersections are controlled by lattices. The second arises from the local calculus of manifolds, based on the study of singularities by Mather [4]. According to Morse theory [5], it enables us to describe cobordisms explicitly in terms of singularities of certain functions.

Using these categories, we give a formal definition to our cobordisms and obtain a set of generators. We also see how it is related to the notion of spherical categories.

[1] J. C. Baez and J. Dolan, Higher-dimensional algebra and topological quantum field theory. *Journal of Mathematical Physics*, 36(11):6073–6105, 1995. http://arxiv.org/abs/q-alg/9503002.
[2] A. Joyal and R. Street, The geometry of tensor calculus I. *Advances in Mathematics*, 88(1):55–112, 1991.
[3] J. Lurie, On the classification of topological field theories. arXiv:0905.0465, 2009.
[4] J. N. Mather. Stability of $C^\infty$ mappings. III: Finitely determined map-germs. *Publications Mathématiques de l'IHÉS*, 35:127–156, 1968.
[5] J. W. Milnor. *Morse Theory.* Number 51 in Annals of Mathematics Studies. Princeton University Press, 1963.



**Restaurants/Pubs:**
There are many wonderful restaurants within walking distance of campus. You can either head East down Spring Garden, North up to the Quinpool area, or the waterfront. Both areas have many choices in terms of ethnicity, cost, etc. Some favourites:
1. Coburg Coffee House (cafe very close to campus, free wifi): 6085 Coburg Rd, (902)429-2326,
2. Wild Leek (vegan): 2156 Windsor St, (902)444-5466,
3. Wasabi House (sushi): 6403 Quinpool Road, (902)429-3300,
4. Mezza (Lebanese): 6386 Quinpool Road, (902) 444-3914,
5. Heartwood (vegetarian/vegan): 6250 Quinpool Road, (902)425-2808
6. Stillwell (Craft beer and snacks): 1672 Barrington St, (902) 421-1672. They also have a popup beer garden just across from the Lord Nelson!
7. Gahan House (Craft beer and quality pub food): 1869 Upper Water St, (902)444-3060
8. McKelvies (seafood): 1680 Lower Water St, (902)421-6161
9. Murphy's (seafood): 1751 Lower Water St, (902)420-1015
10. Salty's (seafood): 1877 Upper Water St, (902)423-6818
11. Pete's Frootique (great to get fresh healthy food to go): 1515 Dresden Row, (902)425-5700
12. Economy Shoe Shop (unique bar/restaurant): 1663 Argyle St, (902)423-8845
13. Henry House (local craft beer and quality pub food): 1222 Barrington St, (902)423-5660
14. The Stubborn Goat (gastropub): 1579 Grafton St, (902)405-4554
15. The Bicycle Thief (quality Italian food): 1475 Lower Water St, (902)425-7993
16. The Old Triangle(Irish pub with food and live music): 5136 Prince St, (902)492-4900
17. Brooklyn Warehouse: 2975 Windsor St, (902)446-8181. Not cheap but original and good.
18. The Charm School: weird little brewpub at 6041 North St, snack menu only.
19. Freeman's Little New York: 6092, Quinpool Road, (902) 429-0241 Good pizza and beer.

**Getting Back To The Airport**

The Stanfield International Airport is a long way out of town, and a taxi there can be quite expensive (more than sixty dollars.) One solution is to share one. Allow half an hour.

If you aren't in a huge hurry, the Metro X bus goes out to the airport for only $3.50 exact change (or a bus ticket and a dollar.) Normally it leaves from Albermarle St (just east of the Citadel) at 20 minutes past the hour. However, bridge restoration means that it only leaves Halifax from 0420 to 1820: evening runs currently leave from the Dartmouth Bridge Terminal (a fairly cheap taxi ride, or on many bus routes) at 1936 to 2326. Allow 50 minutes from Albermarle St, half an hour from the Bridge Terminal.

For details on routes affected by bridge restoration, see
http://www.halifax.ca/transit/bridgeredecking.php

East Shuttle (http://www.eastshuttle.com/) is significantly cheaper than the ordinary taxi for one or two people. Book at least 24 hours in advance.

The organizers wish you a safe flight home!