Geometrical modeling of the heart

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Under the supervision of

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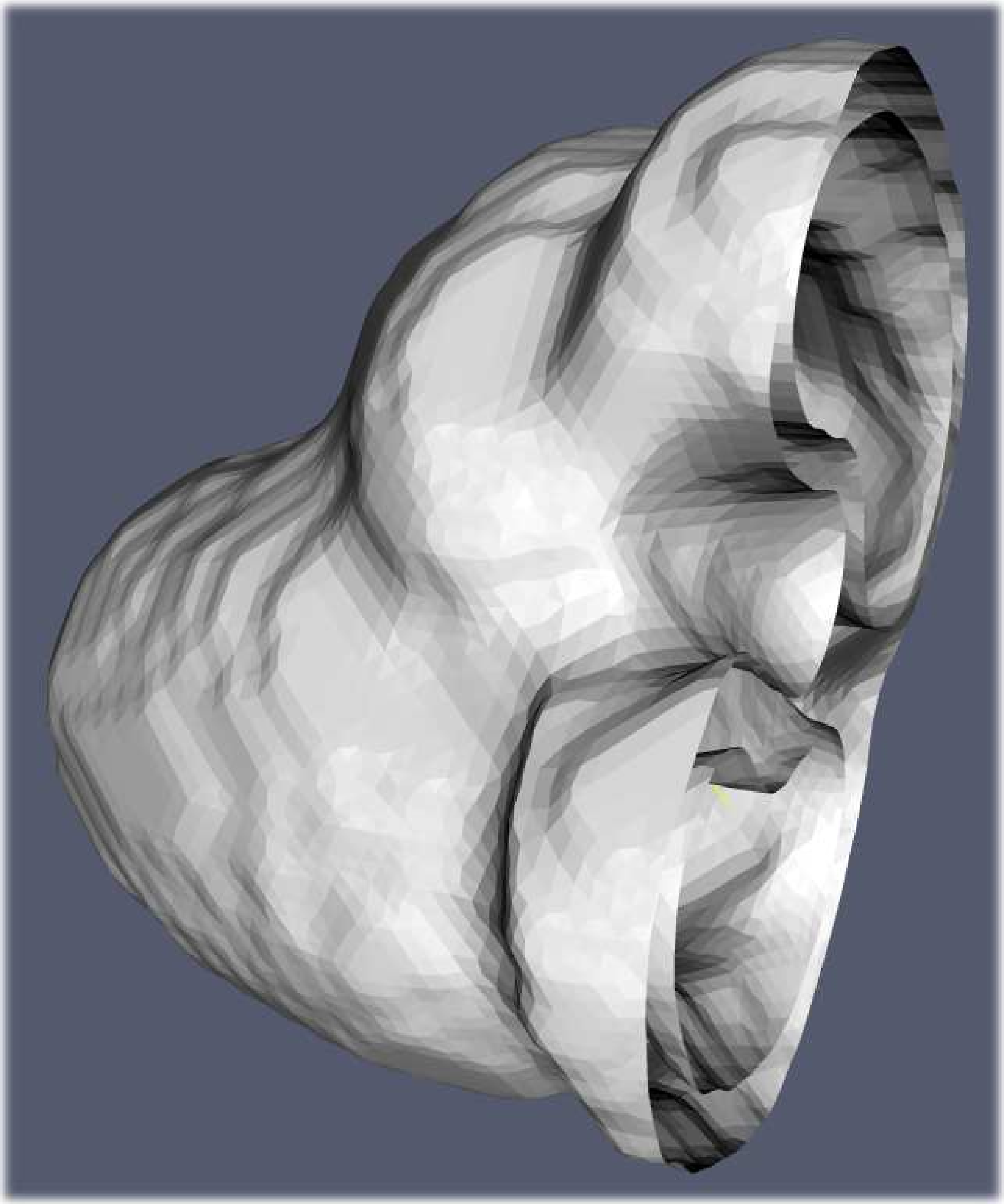


Goal

Creation of a precise 3D model of the heart.

Applications

Numerical calculations:

1. Dynamic of the blood flow
2. Electrophysiology

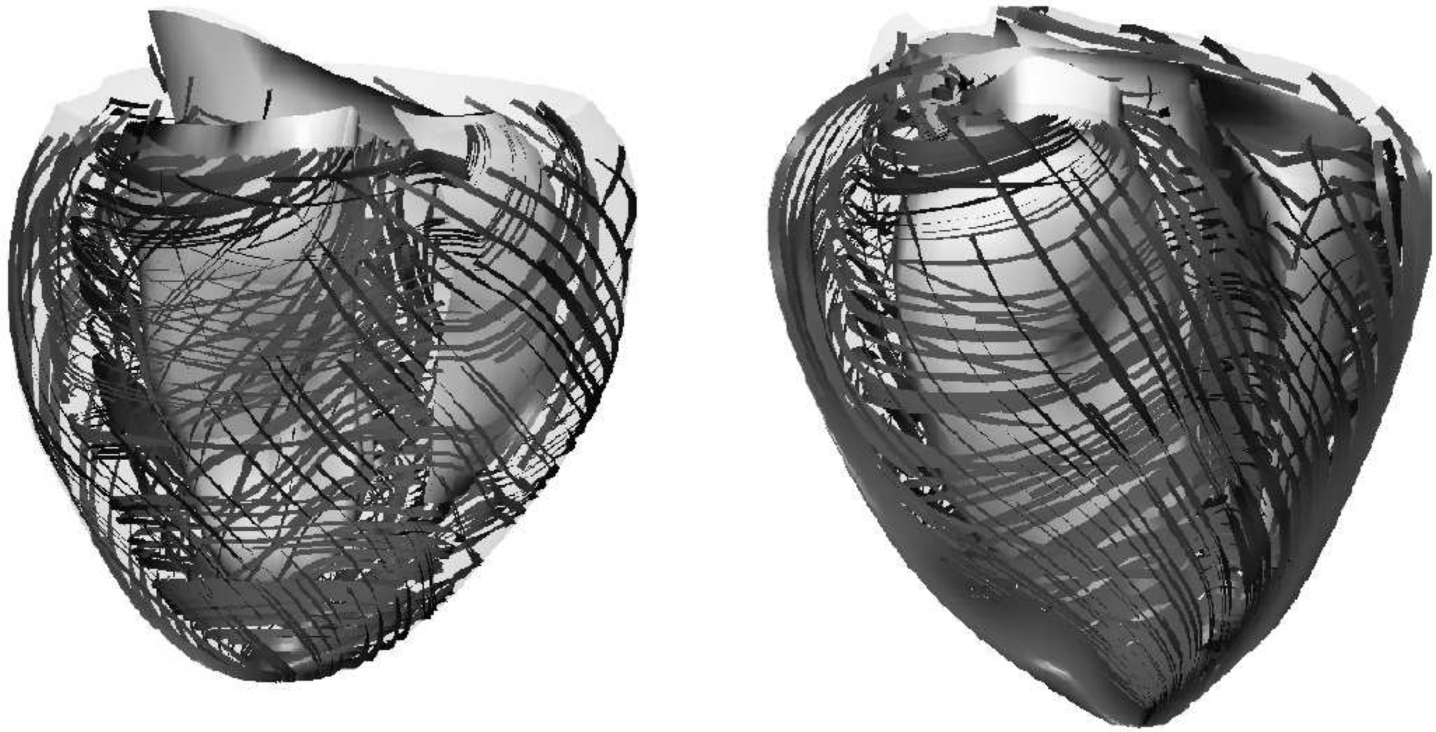
Segmentation Problem

Identify significant regions and boundaries.

Fine Anatomic Details

We are concerned with finding very fine anatomic details such as

1. Orientations of the fibers
2. Geometry of the valves



Fibers of canine and porcine heart.

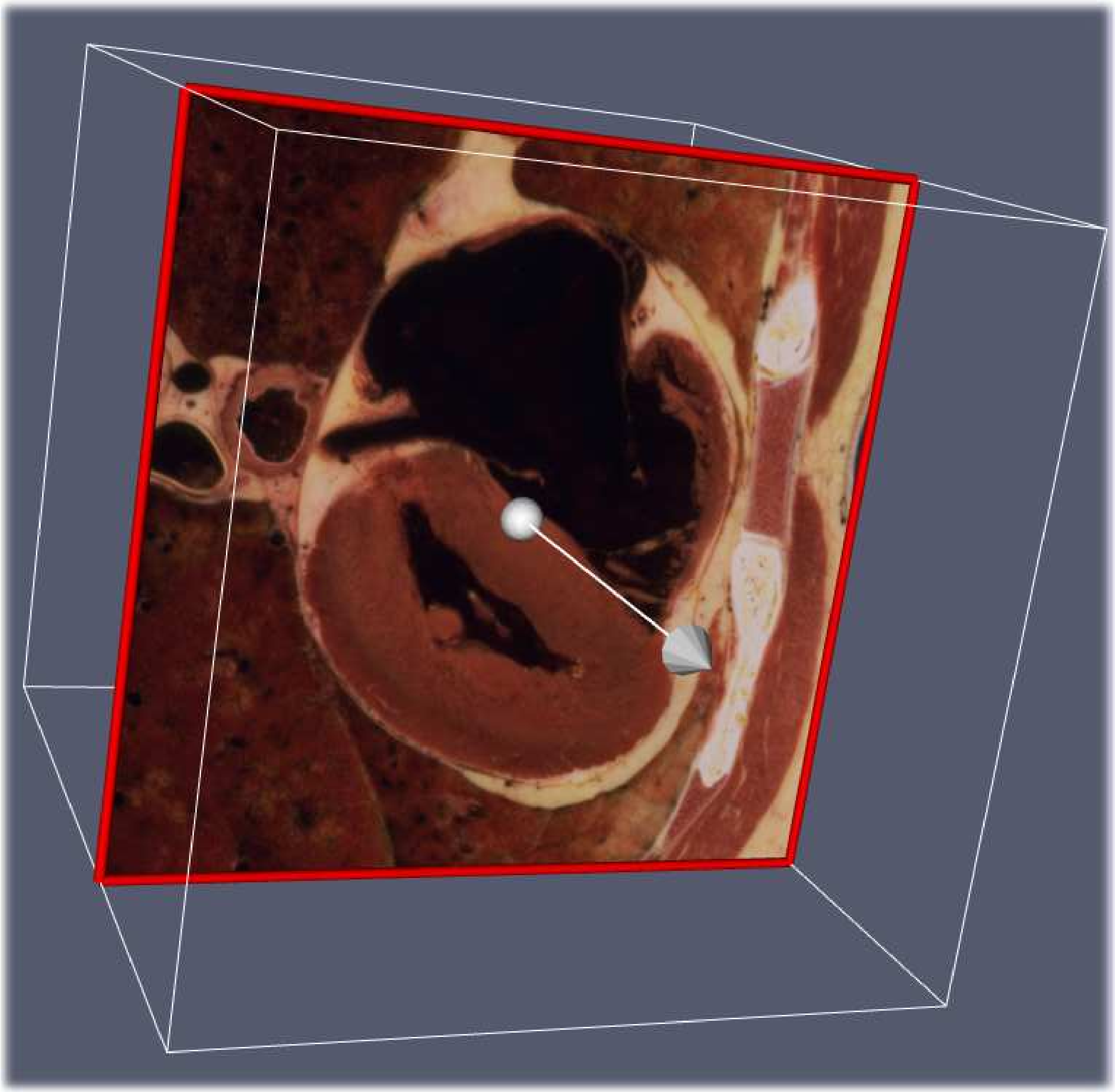
(Hunter & al., Multiscale computational modeling of the heart, *Acta Numerica* (2004))

Data

Visible Human Project

`http://www.nlm.nih.gov/research/visible/visible_human.html`

They created complete and anatomically detailed, representations of the normal male and female human bodies.



Mumford-Shah Functional

Image: $g : \Omega \subseteq \mathbb{R}^N \longrightarrow [0, 1]$.

Mumford-Shah functional:

$$F(u) = \int_\Omega |\nabla u|^2 + \int_\Omega (u - g)^2 + \mathcal{H}^{N-1}(S_u)$$

$u \in SBV(\Omega)$

S_u : set of discontinuities of u

$\mathcal{H}^{N-1}$: $(N-1)$-Hausdorff measure.

Functions of Bounded Variation

A function $u \in L^1(\Omega)$ is in $BV(\Omega)$ iff

$$\sup_{\phi \in C_c^\infty(\Omega)} \int_\Omega u \operatorname{div} \phi < \infty.$$

$$Du = \nabla u \mathcal{L}^N + (u^+ - u^-)\nu_u \mathcal{H}^{N-1} + D^c u$$

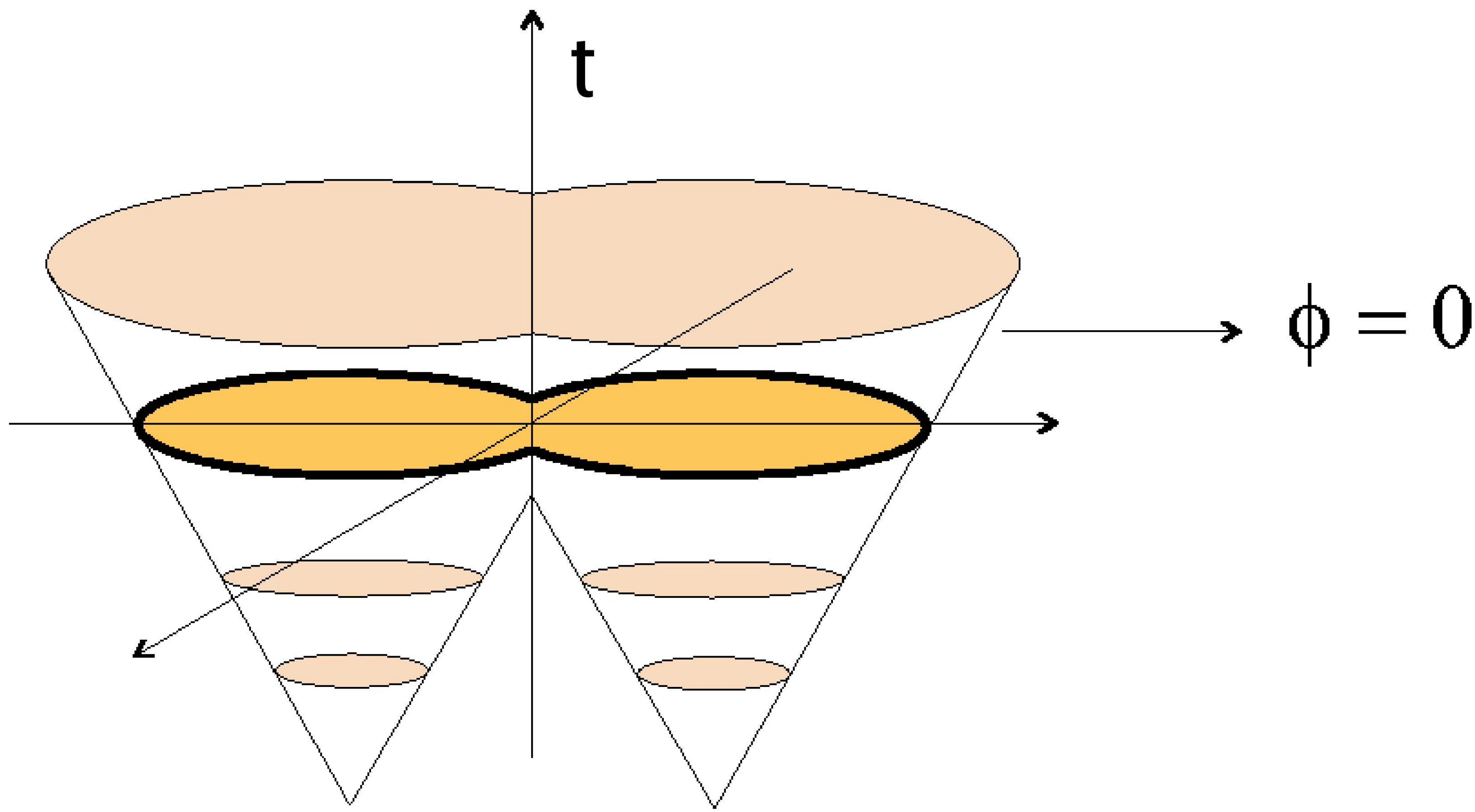
Level Set Method

Let a curve evolve until it stops on the boundaries.

The curve is described by

$$C_t = \{x \;:\; \phi(x,t) = 0\}$$

$$\begin{cases} \frac{d\phi}{dt} = V(x,\phi) \\ \phi(x,0) = \phi_0(x) \end{cases}$$


t
φ = 0

Piecewise Constant Mumford-Shah

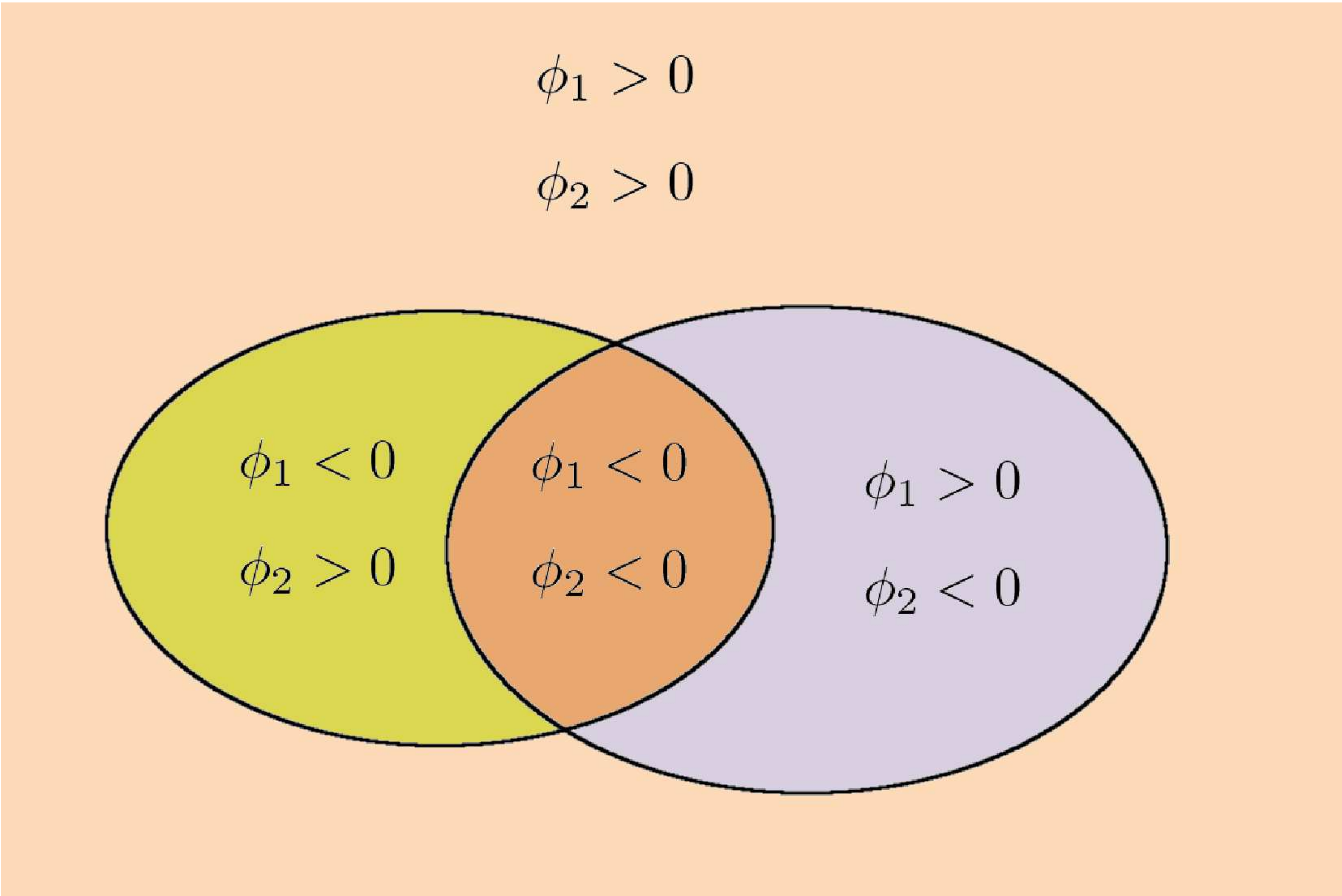
Minimize over binary functions, with image in $\{c_1, c_2\}$. H: heavy side function.

$$F(u, c_1, c_2) = \int_\Omega H(\phi)(g - c_1)^2 + \int_\Omega (1 - H(\phi))(g - c_2)^2 + \int_\Omega H'(\phi).$$

$$c_1 = \int_\Omega H(\phi) g, \qquad c_2 = \int_\Omega (1 - H(\phi)) g.$$

$$\frac{d\phi}{dt} = H'(\phi)\left[\operatorname{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - (g - c_1)^2 + (g - c_2)^2\right]$$

Let n curves evolve to separate the domain in 2^n regions.



Algorithm

At each time step:

1. Compute c_1 and c_2
2. Compute ϕ^n from ϕ^{n-1}, c_1 and c_2
3. Possibly reinitialize ϕ.
4. Stop if $Error < \epsilon$.

Software

Insight Toolkit (www.itk.org)

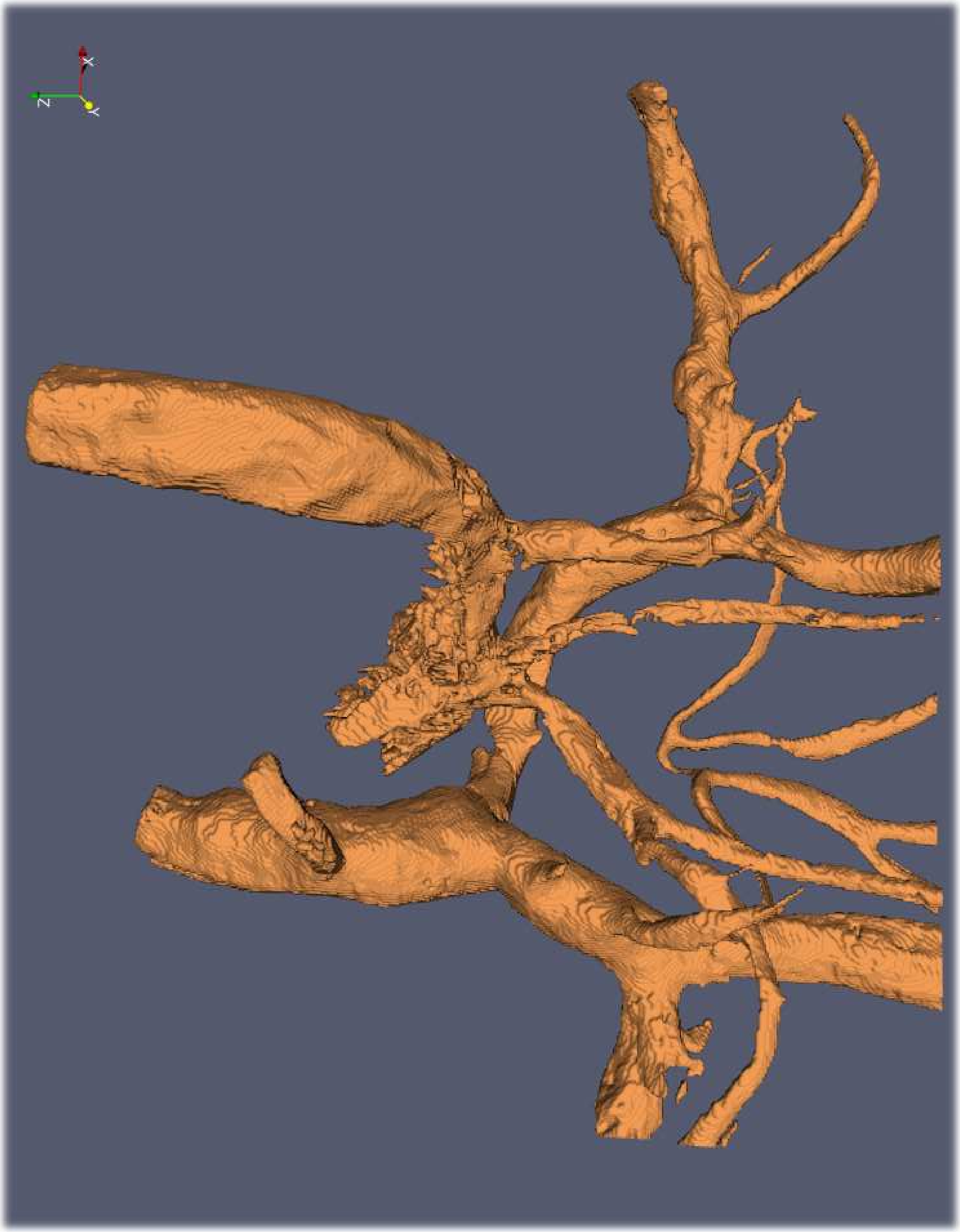
Paraview (www.paraview.org)

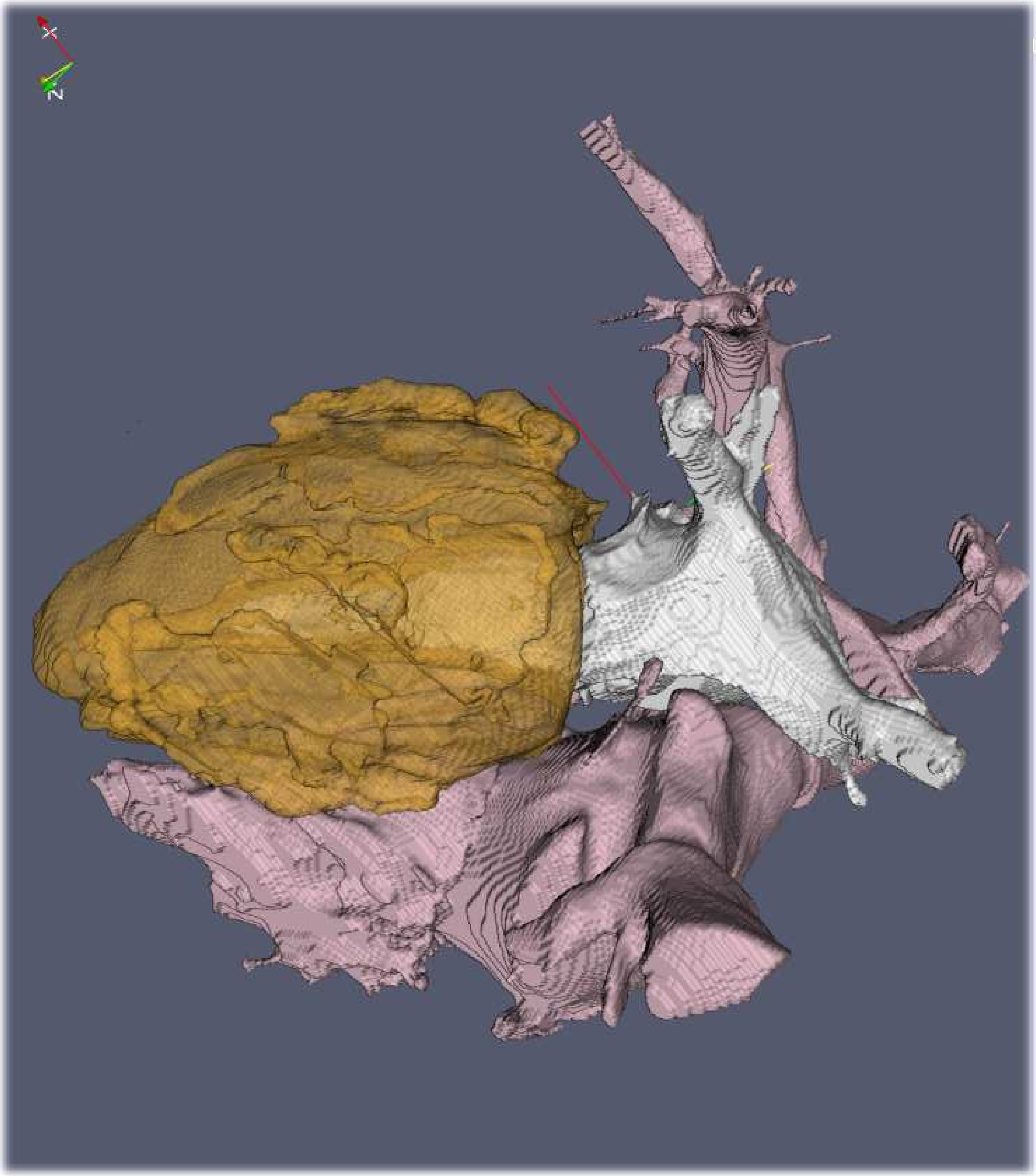
Both are developped by Kitware (www.kitware.com)

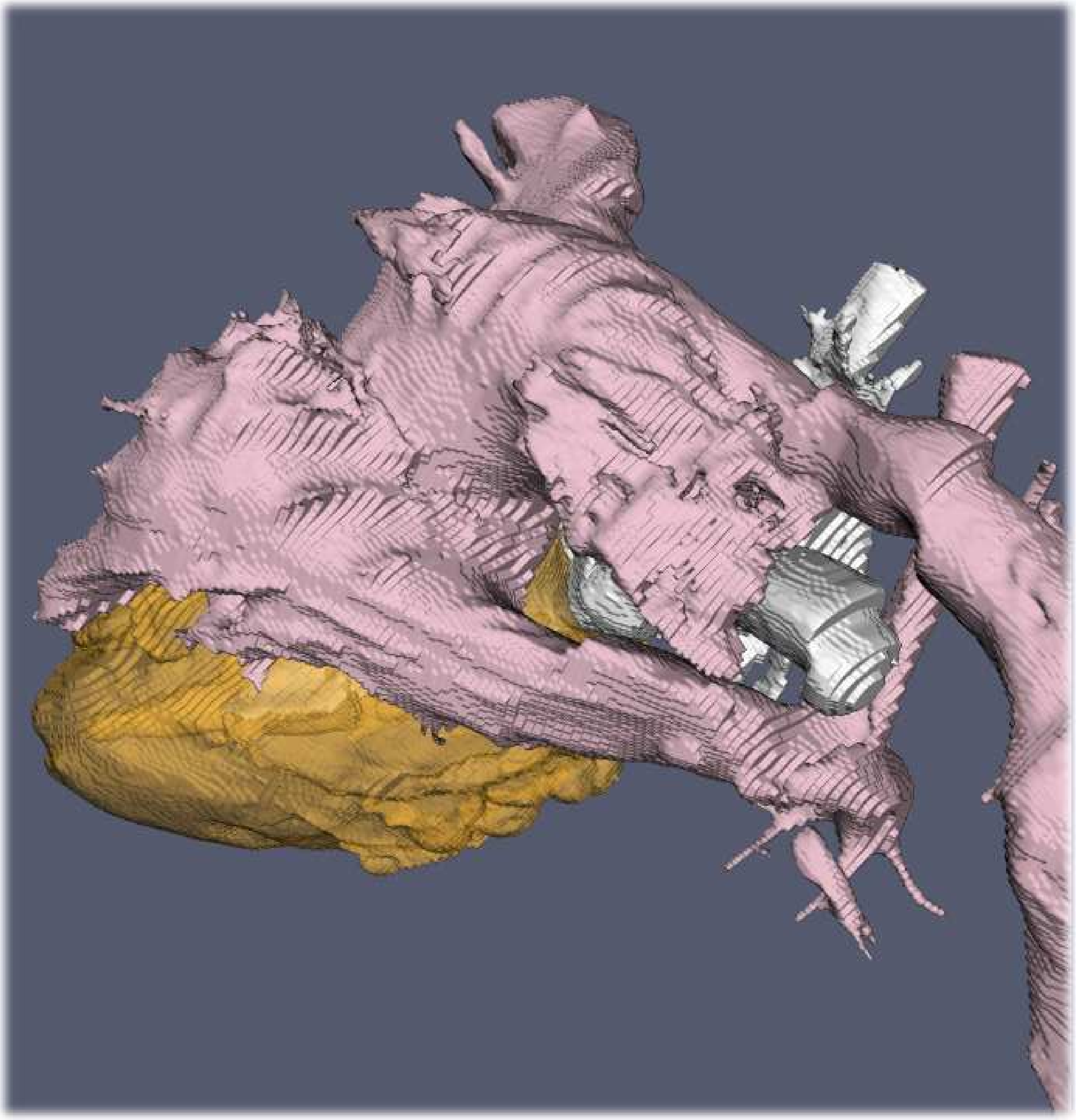
Speed base algorithms

$$V(x, \phi) = u(|\nabla g(x)|)$$

$$u(\nabla g(x)) = \begin{cases} 1, & |\nabla g(x)| \text{ small} \\ 0, & |\nabla g(x)| \text{ large} \end{cases}$$


X
Z
Y





Mumford-Shah (two colours)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Mumford-Shah (four colours)