Improving the Diversity Product of Space-Time Hamiltonian Constellations

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Abstract — **This paper proposes new unitary space-time constellation designs with high diversity products for any number of antennas and any rate based on Slepian's group codes. Many of our Hamiltonian and product constellations have the best known diversity products in the literature, and outperform all other constellation designs.**

I. Introduction

Consider multiple antennas in a Rayleigh flat-fading channel with M transmitter antennas and N receiver antennas. Let $\mathcal{V} = \{V_l\}_{l=0}^{L-1}$ be an $M \times M$ unitary constellation. The rate is $R = \log_2 L/M$. The design problem of differential unitary space-time constellation [1] is to maximize the diversity product, $\zeta_{\mathcal{V}}$ as:

$$\zeta_{\mathcal{V}} = \frac{1}{2} \min_{0 \le l < l' \le L-1} |\det(V_l - V_{l'})|^{\frac{1}{M}}, \tag{1}$$

$0 \le \zeta_{\mathcal{V}} \le 1$. A constellation $\mathcal{V}$ which has $\zeta_{\mathcal{V}} > 0$ is said to have *full diversity*. Our goal in this paper is to find a set $\mathcal{V}$ of $M \times M$ unitary matrices which has $\zeta_{\mathcal{V}}$ as large as possible. From [2], the diversity product of a 2×2 Hamiltonian matrix equals one half of the Euclidean distance between two points in $\mathbb{C}^2$. Using a transformation from $\mathbb{R}^4$ to $\mathbb{C}^2$, the problem of constructing the 2×2 Hamiltonian constellation is reduced to finding L points in $\mathbb{R}^4$ such that the minimum distance between any two points is as large as possible.

II. Design of Unitary Constellations

A. Hamiltonian constellation: We use an $(L, 4)$ cyclic group code, $\{X_l\}_{l=0}^{L-1} = O_l X$; $O_l = \mathrm{diag}(A(lk_1), A(lk_2))$ where

$$A(k_i) = \begin{bmatrix} \cos\frac{2\pi}{L}k_i & \sin\frac{2\pi}{L}k_i \\ -\sin\frac{2\pi}{L}k_i & \cos\frac{2\pi}{L}k_i \end{bmatrix}, \quad k_i \in \{1, 2, \ldots, L-1\}, \tag{2}$$

and an initial vector $X = (\sqrt{x_1}, 0, \sqrt{x_2}, 0)$ where $x_1 + x_2 = 1$ and $x_1, x_2 \ge 0$ to get L points on a unit sphere in $\mathbb{R}^4$. The $M \times M$ Hamiltonian constellation $\mathcal{H} = \{J_l\}_{l=0}^{L-1}$ has the block diagonal form

$$\begin{aligned} J_l &= \mathrm{diag}(H_l^{1,2}, H_l^{3,4}, \ldots, H_l^{M-1,M}) \quad \text{for } M \text{ even} \quad (3) \\ &= \mathrm{diag}(e^{j2\pi k_1 l}, H_l^{2,3}, \ldots, H_l^{M-1,M}) \quad \text{for } M \text{ odd} \quad (4) \end{aligned}$$

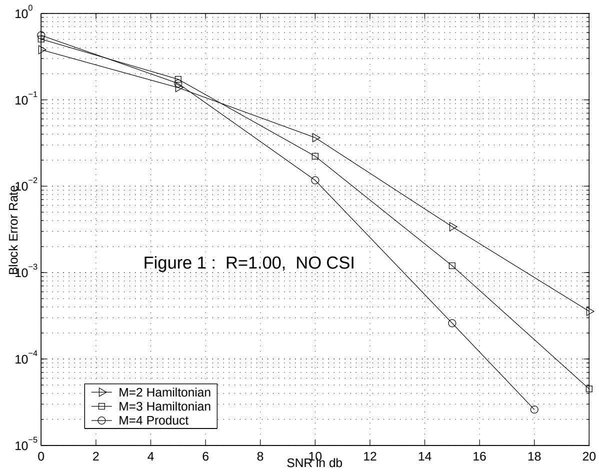
where $H_l^{m,n}$ is defined as

$$H_l^{m,n} = \begin{bmatrix} \sqrt{x_1} e^{-j\frac{l2\pi k_m}{L}} & -\sqrt{x_2} e^{j\frac{l2\pi k_n}{L}} \\ \sqrt{x_2} e^{-j\frac{l2\pi k_n}{L}} & \sqrt{x_1} e^{j\frac{l2\pi k_m}{L}} \end{bmatrix}. \tag{5}$$

B. Product Constellations: A product constellation $\mathcal{P}$ is constructed by a product of Hamiltonian constellation $\mathcal{H} = \{J_l\}_{l=0}^{L_H-1}$ as in (3) and (4) for M even and odd respectively and a cyclic group $\{O_k\}_{k=0}^{L_C-1}$, where $O_k = \mathrm{diag}(e^{j2\pi r_1 k/L_C}, e^{j2\pi r_2 k/L_C}, \ldots, e^{j2\pi r_M k/L_C})$, as $\mathcal{P} = \{J_l O_k\}_{l,k=0}^{L_H-1, L_C-1}$. When the case where M is odd, we propose another unitary product constellation, $\mathcal{P}_{\mathcal{H}}$, which is obtained by a product of two Hamiltonian constellations with the diagonal blocks in different order, as $\mathcal{P}_{\mathcal{H}} = \{J_l J_k^\dagger\}_{l,k=0}^{L_{H_1}-1, L_{H_2}-1}$ where $J_k^\dagger$ denotes a block diagonal matrix with different order of J_k in (4) as $J_k^\dagger = \mathrm{diag}(H_k^{1,2}, H_k^{3,4}, \ldots, H_k^{M-2,M-1}, e^{j2\pi r_M k/L_{H_2}})$. Although $\mathcal{H}$, $\mathcal{P}$ and $\mathcal{P}_{\mathcal{H}}$ do not form groups, the optimization of $\zeta_{\mathcal{V}}$ requires checking only $L-1$ distinct matrices which makes it comparable to those codes that use group constellations.

III. Results and Performance

Hamiltonian constellations of $L = 2$ to 5 for M even case are optimal constellations whose diversity product achieve the theoretical upper bound as given by $\sqrt{\frac{L}{2(L-1)}}$. Figure 1 shows the block error rate performance of proposed Hamiltonian and product constellations at $R = 1.00$ for $M = 2, 3, 4$ and $N = 1$ of $\mathcal{H}_{2\times2}, \mathcal{H}_{3\times3}$ and $\mathcal{P}_{4\times4}$ respectively.



References

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[2] A. Shokrollahi, B. Hassibi, B.M. Hochwald and W. Sweldens, "Representation theory for high-rate multiple-antenna code designs," *IEEE Trans. Info. Theory*, vol. 47, pp. 2335-2367, Sept 2001.

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