**NSERC USRA Summer 2019**

Professor: Hadi Salmasian

**First Project**

**Title:** A combinatorial/probabilistic experiment with a variation of Solitaire.

**Brief overview:** An $n$-variable polynomial $f(x_1, \ldots, x_n)$ is called symmetric if it does not change by any permutation of its variables. Some simple examples of symmetric polynomials are the sum $x_1 + \cdots + x_n$, or more generally the power sums $x_1^k + \cdots + x_n^k$.

The $n$-variable symmetric polynomials form a ring. The structure of this ring has deep connections with algebra, geometry, and combinatorics. For example, a distinguished basis of this ring (the Schur polynomials), that is indexed by certain combinatorial objects called Young diagrams, is related to character theory of the symmetric group $S_n$, representation theory of the group $GL(n, \mathbb{C})$ of invertible $n \times n$ matrices, and cohomology classes of Grassmannian manifolds.

Since the Schur polynomials form a basis for the ring of n-variable symmetric functions, the product of two Schur polynomials can be expressed as a linear combination of finitely many Schur polynomials. The coefficients that appear in these linear combinations (Littlewood-Richardson coefficients) are extremely important. One can give a formula for the computation of these coefficients in terms of combinatorial operations on Young diagrams. This gives rise to a beautiful combinatorial theory with numerous connections to different areas of mathematics.

The Schur polynomials have important generalizations, the so-called *Jack* and *Macdonald* polynomials, that play a significant role in studying symmetric spaces. The Jack polynomials are also connected to quantum physics: they are the eigenstates of the Hamiltonian of the *quantum n-body problem.*

The goal of this research project is to investigate problems about the aforementioned polynomials. The problems that will be investigated will be of combinatorial and algebraic nature. For example, one important question is to determine to what extent the properties of Littlewood-Richardson coefficients extend to similar coefficients that appear for Jack and Macdonald polynomials and their variants? Some of the questions can lead to research papers that are publishable in peer-reviewed journals.

To be eligible for this project, the student should have a solid background in undergraduate algebra (the theory of groups, rings, and fields).