University of Ottawa

Department of Mathematics and Statistics

MAT 2348:
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Homework 1 – January 26, 2012

Surname ______________________ First Name ______________________

Student # ______________________

ONLY QUESTION 9 NEEDS A COMPLETE SOLUTION. FOR THE REST OF THE PROBLEMS JUST WRITE DOWN THE ANSWER, WITHOUT JUSTIFICATIONS! For a set X we denoted by $P(X)$ the collection of all subsets of X.

1. Find the following sets $\bigcap_{t\in\mathbf{R}}[\frac{1}{t^2+1}, 3+(t-5)^2]$ and $\bigcup_{t\in\mathbf{R}}[\frac{1}{t^2+1}, 3+(t-5)^2]$. In each case the answer is an interval!

2. Are the following statements true for all sets A, X, Y? Answer by YES or NO!
(a) $A \subseteq X$ and $A \subseteq Y \Rightarrow A \subseteq X \bigcap Y$
(b) $A \subseteq X \bigcup Y \Rightarrow$ either $A \subseteq X$ or $A \subseteq Y$
(c) $X \bigcap Y \subseteq A \Rightarrow$ either $X \subseteq A$ or $Y \subseteq A$
(d) $A \backslash (X \bigcup Y) \subseteq A \backslash Y$
(e) $A \Delta (X \bigcup Y) = (A \Delta X) \bigcup (A \Delta Y)$

3. Let $f : \mathbf{R} \to \mathbf{Z}$ be the function given by $f(x) =$ the smallest integer $n \in \mathbf{Z}$ such that $3x < n$. So, $f(\frac{-4}{5}) = -2$. Which of the following statments are true?
(a) f is injective, but not bijective
(b) f is onto, but not bijective
(c) f is bijective
(d) f is not $1-1$ and f is not onto
(e) f is bijective, but it is not onto

4. Consider the function in exercise 3.
(a) Find $f([-1, 1])$
(b) Find $f^{-1}(\{2, 3, 4\})$

5. Consider the function in exercise 3. Let $g : \mathbf{Z} \to \mathbf{R}$ be the function given by $g(n) = \frac{n}{3}$ for all $n \in \mathbf{Z}$.
(a) Find the domain and codomain of $f \circ g$
(b) For every x in the domain of $f \circ g$ find a formula for $f \circ g(x)$

6. Let $\mathbf{N} = \{0, 1, 2, 3, \dots\}$ and let $f : \mathbf{N} \to P(\mathbf{N})$ be the function given by $f(n) = \{x \in \mathbf{N} | x < n\}$ for all $n \in \mathbf{N}$. Which of the following statements are true?
(a) f is injective, but not bijective
(b) f is onto, but not bijective
(c) f is bijective
(d) f is not $1-1$ and f is not onto
(e) f is bijective, but it is not onto

7. Below you will find 7 tentatives to define a function. In each case indicate if one can or not define a function. Answer by YES or NO. We denote by B^A the set of functions defined on A with values on B.

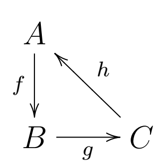
(a) $f : \mathbf{N} \to \mathbf{R}$, $f(n) =$ the number $y \in \mathbf{R}$ such that $y^2 + ny - 1 = 0$
(b) $f : \mathbf{N} \to \mathbf{R}$, $f(n) =$ the smallest number $y \in \mathbf{R}$ such that $y^2 + ny - 1 = 0$
(c) $f : \mathbf{N} \to [0, \infty)$, $f(n) =$ the smallest number $y \in \mathbf{R}$ such that $y^2 + ny - 1 = 0$
(d) $f : \mathbf{N} \to \mathbf{R}$, $f(n) =$ the largest number $y \in \mathbf{R}$ such that $y^2 + ny - 1 < 0$
(e) $f : \mathbf{N} \to \mathbf{R}$, $f(n) =$ the number $y \in \mathbf{R}$ such that $(n-1)y = n + 1$
(f) $\Phi : \mathbf{R}^{\mathbf{N}} \to \mathbf{R}$, $\Phi(f) = f(3)$
(g) $\Phi : \mathbf{R}^{\mathbf{N}} \to \mathbf{N}$, $\Phi(f) = f(3)$

8. Consider the functions $f : \mathbf{N} \to P(\mathbf{N})$ given by $f(n) = \{x \in \mathbf{N} | x < n\}$ for all $n \in \mathbf{N}$ and $g : P(\mathbf{N}) \to \mathbf{N}$ given by $g(A) = |A|$ if A is finite and $g(A) = 0$ if A is not finite, for any subset A of $\mathbf{N}$.

Indicate which of the following statements are true.

(a) $g \circ f = 1_{\mathbf{N}}$ and $f \circ g = 1_{P(\mathbf{N})}$
(b) $f \circ g = 1_{\mathbf{N}}$ and $g \circ f = 1_{P(\mathbf{N})}$
(c) $g \circ f = 1_{\mathbf{N}}$ and $f \circ g$ is not injective and g is surjective
(d) $f \circ g \circ f$ is injective and $g \circ f \circ g$ is not onto
(e) $f \circ g \circ f$ is not injective and $g \circ f \circ g$ is onto

9. Let A, B, C be sets and f, g, h be functions as in the following diagram :



Assume that the following 3 functions $h \circ g \circ f : A \to A$, $f \circ h \circ g : B \to B$, $g \circ f \circ h : C \to C$ are bijective. Show that f, g, h are bijective. Hint: Think about the associativity of composition, think about exercise (and its proof done in class) 8 on page 288.

More space for exercise 9