MAT 1322

SAMPLE EXAMINATION

•Time: 3 hours

•Calculators are permitted. (Non-programmable, non-graphing, no differentiation or integration capability.) Notes or books are not permitted.

•Work the problems in the space provided. Use the back-pages for rough work if necessary. Do not use any other paper. Show all work.

•Circle the correct answers for multiple choice problems. Numerical answers are rounded. Work on multiple choice problems will be examined in case of suspected fraud.

1. [4 points] The finite region bounded by curves $y = x^2$ and $y = 4$ is rotated about the line $y = -2$. Sketch the region. Find the volume of the resulting solid.

Work.

2. [4 points] An object is taken from an oven at a temperature of $300\,^\circ$C to a room at $20\,^\circ$C. Its temperature $u = u(t)$ then decreases according to Newton's law of cooling $\frac{du}{dt} = k(u - 20)$. After 15 minutes its temperature is $200\,^\circ$C.

(a)Find the temperature $u(t) =$ $\boxed{}$

(b)When will the temperature reach $100\,^\circ$C? $t =$ $\boxed{}$

(c)Sketch the graph of $u(t)$ showing the values for $t = 0$ and as $t \to \infty$.

Work.

3. [4 points] Determine if the following series are convergent or divergent and state the name of the test you used for that purpose. (Record you answer next to the series below.)

		conv./div.	test used
(a)	$\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$		
(b)	$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n^3+5)^{1/3}}$		

Work.

4. [4 points] Let $f(x, y) = \dfrac{x^2}{4} + \dfrac{y^2}{9}$.

(a)Sketch at least three labeled level curves of f and the gradient vector at a point on each of them. (All in one sketch, on the same axes.)

(b)Find the equation of the tangent plane to the graph of f at the point where $(x, y) = (2, 3)$.

Work.

5. [2 points] Evaluate $\displaystyle\int_0^3 \frac{2}{(x-2)^{4/3}}\,dx$ if possible.

A. -10.76 B. 1.24 C. -1.24 D. 2.40 E. -2.40 F. divergent

6. [2 points] Find the area of the region enclosed by the curves $y = x$ and $y = 7x - 3x^2$.

A. 2 B. 4 C. 6 D. 8 E. 9 F. 11

7. [2 points] Consider the initial value problem $y' = x + y$, $y(0) = 1$. Use Euler's method with step size 0.1 to approximate $y(0.2)$.

A. 1.2000 B. 1.1000 C. 1.2200 D. 1.2300 E. 1.0100 F. 1.0200

8. [2 points] Solve the initial value problem $\frac{dy}{dx} = \frac{y}{1+x^2}$, $y(0) = 2$ to find $y(1)$.

A. 2.19 B. 2.00 C. 4.39 D. 4.00 E. 3.14 F. undefined

9. [2 points] Let $f(x) = e^{x^3}$. Calculate the 9th derivative $f^{(9)}(0)$ using the exponential series and Taylor's formula.

A. 0 B. 350 C. 810 D. 3490 E. 60480 F. 78520

10. [2 points] If $(1 + x^3)^{0.6} = \sum_{n=0}^{\infty} b_n x^n$ then $b_3 =$

A. -1.00 B. -0.05 C. 0.00 D. 0.05 E. 0.60 F. 1.00

11. [2 points] Find the radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n (x+3)^n}{n}$.

A. 1 B. 5 C. 1/5 D. 3 E. 1/3 F. ∞

12. [2 points] Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 3^{n+1}}{7^n}$.

A. divergent B. 3.15 C. 1.40 D. 7.35 E. 6.65 F. 5.25

13. [2 points] Determine which of the series converge.

$$(1)\ \sum_{n=1}^{\infty} \frac{(-1)^n 100}{n} \qquad (2)\ \sum_{n=1}^{\infty} \frac{n}{5^n} \qquad (3)\ \sum_{n=1}^{\infty} \frac{n^2}{(n+100)^2}$$

A. (1) and (2) B. (1) and (3) C. (2) and (3)
D. (1) only E. (2)only F. (3) only

14. [2 points] *Use the linear approximation* of $f(x, y) = 2x^2y^2 + 3xy + x$ at $(1, 1)$ to approximate $f(0.9, 1.1)$.

A. 5.8302 B. 5.9000 C. 6.1000 D. 6.0302 E. 6.0000 F. 5.9302

15. [2 points] Find the equation of the tangent plane of the surface $z^2+x^2-4xy+y^2=2$ at the point $(1,1,2)$.

A. $2z-x-y=2$ B. $2z+x+y=6$ C. $z+x+y=4$

D. $z=2$ E. $2x+2y-z=2$ F. $z-x-y=0$

16. [2 points] Let $f(x,y,z)=2x^2y^2z+4xy^3z^2$. Find $f_{xyz}(2,1,1)$.

A. 28 B. 32 C. 36 D. 24 E. 38 F. 40

17. [2 points] Find the directional derivative of $f(x, y) = x^2y + 4y^2$ at the point $(2, 1)$ in the direction of the vector $\langle 1, \sqrt{3} \rangle$.

A. $2 - 6\sqrt{3}$ B. $2 + 3\sqrt{3}$ C. $2 - 3\sqrt{3}$

D. $2 + 6\sqrt{3}$ E. $4 + 12\sqrt{3}$ F. $2 - \sqrt{3}$

18. [2 points] Suppose $z = f(x, y)$ where $x = g(t)$ and $y = h(t)$. Given the data

$g(1) = 1,\ g'(1) = 2,$
$h(1) = 2, h'(1) = 3,$
$f_x(1, 2) = -1,\ f_y(1, 2) = 2.$

Find $\frac{dz}{dt}$ when $t = 1$.

A. -1 B. 1 C. 4 D. -4 E. 8 F. impossible