# University of Ottawa
# MAT 2377 Midterm Exam

**June 4, 2009. Duration: 110 minutes. Professor: Catalin Rada**

Family Name:______________________

First Name:_______________________

- You have 110 minutes to complete this exam.
- This is a open book exam. The use of laptops, cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed. The last question is a bonus question, and it worths 2 points. To get the 2 extra points you have to provide a solution. Any other problem worths 1 point.
- Good Luck!

Student number:____________________, Total marks: ___________ out of 13

| Question | Score | Record your answer here |
|---|---|---|
| 1 | | |
| 2 | | |
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| Total | | |

- (1) In a factory the probability that a certain item is being defective is $p = 0.05$. An inspector selects 6 items at random. Let $X$ be the number of defective items in the sample. Find $P(X \geq 2)$.

  A) 0.042252 B) 0.044561 C) 0.032775 D) 0.034670 E) none of the preceding

  **Solution:** Since $X \sim$ Binomial $(6, 005)$ we do have $P(X \geq 2) = 1 - P(X < 2) = 1 - \{P(X = 0) + P(X = 1)\} = 1 - \{\binom{6}{0}(0.05)^0(0.95)^6 + \binom{6}{1}(0.05)^1(0.95)^5\} = 1 - \{(0.95)^6 + 6(0.05)(0.95)^5\} = 1 - \{0.735091 + 0.232134\} = 1 - 0.967225 = 0.032775$, so we choose C.

- (2) Telephone calls enter a college switchboard on the average of 2 every 3 minutes. Let $X$ be the number of calls in a 9 minute period. Compute $P(X \geq 5)$. We assume that the calls are modeled by a Poisson process.

  A) 0.715 B) 0.815 C) 0.915 D) 0.615 E) none of the preceding

  **Solution:** The new $\lambda = 6$, so $P(X \geq 5) = 1 - P(X \leq 4) = 1 - \sum_{k=0}^{4} \frac{e^{-6}6^k}{k!} = 1 - e^{-6}\{1 + 6 + 18 + 36 + \frac{6\times6\times6}{4}\} = 1 - e^{-6}\{115\} = 0.715$, so we choose A.

- (3) For an IQ test on planet MathematiX, the results are normally distributed with a mean of 100 points and a standard deviation of 15 points. What is the probability that a person chosen randomly will have an IQ score between 70 and 130?

  A) 0.9215 B) 0.9105 C) 0.9155 D) 0.9545 E) none of the preceding

  **Solution:** We compute $P(70 < X < 130) = P(\frac{70-100}{15} < Z < \frac{130-100}{15}) = P(-2 < Z < 2) = \Phi(2) - \Phi(-2) = 0.977250 - 0.022750 = 0.9545$, where $Z$ is standard normal; so we choose D.

- (4) An electronic switching device occasionally malfunctions and needs to be replaced. It is known that the device is satisfactory if it makes, on the average, no more than 0.20 error per hour. A particular 5-hour period is chosen as a "'test"' on the device. If no more than 1 error occurs, the device is considered satisfactory. What is the probability that a satisfactory device will be considered unsatisfactory on the basis of this test? Assume that the errors occur according to a Poisson process.

  A) 0.2435 B) 0.2642 C) 0.3568 D) 0.3680 E) none of the preceding

  **Solution:** Let $X =$ "number of errors in 5 hours". Assuming that the device is satisfactory, i.e., 0.20 error per hour, then $X$ follows a Poisson distribution with $\lambda = (.2)(5) = 1$. The probability that a satisfactory device will be considered unsatisfactory is $P(X > 1) = 1 - P(X \leq 1) = 1 - \{e^{-1}\frac{1^0}{0!} + e^{-1}\frac{1^1}{1!}\} = 0.2642$, so we choose B.

- (5) Assume that when a chip of a certain type is subjected to an accelerated life test, the lifetime, $X$ (in weeks) has a gamma distribution with mean 24 weeks and standard deviation 12 weeks. Find $P(12 < X < 24)$.

  A) 0.42366 B) 0.45322 C)0.36725 D) 0.32826 E) none of the preceding

**Solution:** We know $\frac{r}{\lambda} = 24$ and $\sqrt{\frac{r}{\lambda^2}} = 12$, so $r = 24\lambda$ and $\sqrt{\frac{24\lambda}{\lambda^2}} = 12$, hence $\lambda = 1/6$, and $r = 24 \times \frac{1}{6} = 4$. We have $P(12 < X < 24) = \sum_{k=0}^{3} \frac{e^{-\frac{1}{6}\times 12}((1/6)12)^k}{k!} - \sum_{k=0}^{3} \frac{e^{-\frac{1}{6}\times 24}((1/6)24)^k}{k!} = e^{-2}\{6.33333\} - e^{-4}\{23.66666\} = 0.85712 - 0.43346 = 0.42366$, so we choose A.

- (6) A SugaryCola vending machine is set such that the amount of drink dispensed is a random variable with a mean of 200 liters and a standard deviation of 15 liters. What is the probability that the average amount dispensed in a random sample of size 36 is at least (i.e., greater or equal to 204) 204 liters.

  A) 0.038242 B) 0.052864 C) 0.034754 D) 0.054799 E) none of the preceding

  **Solution:** We compute $P(\overline{X} \geq 204) = P(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \geq \frac{204-200}{\sigma/\sqrt{n}}) = P(Z \geq \frac{4}{15/\sqrt{36}}) = P(Z \geq \frac{4\times 6}{15}) = P(Z \geq 1.16) = 1 - P(Z < 1.16) = 1 - 0.945201 = 0.054799$, where $Z$ is standard normal; so we choose D.

- (7) Eight thousands tickets are to be sold at one dollar each in a lottery. The prize is a $3,000.00$ dollars motorcycle. If I have purchased two tickets, what is my expected gain?

  A)$-1.25$ B) $-1.75$ C) $-2.25$ D)$-2.75$ E) none of the preceding

  **Solution:** The gain, say $X$, can take one out of two values: either I am loosing 2 dollars (i.e., the gain is $-2$ dollars), or I win 2998.00 dollars. The probabilities are $\frac{7998}{8000}$, and $\frac{2}{8000}$, respectively. So, the expectation is $-2 \times \frac{7998}{8000} + 2998.00 \times \frac{2}{8000} = -1.25$ dollars, so we choose A.

- (8) Suppose that a random variable $X$ has p.d.f. $f_X(x) = 2(1-x)$ if $0 < x < 1$, and zero elsewhere. Compute $E(6X + 3X^2)$.

  A) $\frac{11}{2}$ B) $\frac{9}{2}$ C) $\frac{7}{2}$ D)$\frac{5}{2}$ E) none of the preceding

  **Solution:** Note that $E(6X+3X^2) = 6E(X)+3E(X^2)$. We compute $E(X) = \int_0^1 x2(1-x) = 2\{\frac{x^2}{2} - \frac{x^3}{3}|_0^1\} = 1/3$ and $E(X^2) = \int_0^1 x^2 2(1-x) = 2\{\frac{x^3}{3} - \frac{x^4}{4}|_0^1\} = 1/6$. Hence the result is $6 \times \frac{1}{3} + 3 \times \frac{1}{6} = \frac{5}{2}$, so we choose D.

- (9) The chance that an applicant for driver's license passes the road test is 75%. Compute i) the probability that an applicant passes the test on his/her fifth try; ii) the expectation and variance for the number of trials until he/she passes the road test.

  A) i) 0.0333, ii) 1.6666 and 0.4220 B) i) 0.0033, ii) 1.3333 and 0.4500 C) i) 0.0029, ii) 1.3333 and 0.4444 D) i) 0.0230, ii) 1.6666 and 0.4420 E) none of the preceding

  **Solution:** Let $X$ be the number of trials needed to observe the first succes (i.e., to pass the test) it is geometric, $p = 0.75$; i) $P(X = 5) = (0.25)^4(0.75) = 0.0029$; $E(X) = \frac{1}{p} = \frac{1}{0.75} = \frac{4}{3} = 1.3333$ and $Var(X) = \frac{1-p}{p^2} = \frac{0.25}{0.75^2} = 0.4444$. So we choose C.

- (10) Find the probability that more than 30 but less than 35 of the next 50 births at a certain hospital will be boys.

A) 0.04314 B) 0.05689 C) 0.05019 D) 0.04526 E) none of the preceding

**Solution:** Let $X$ be the number of boys, so $X \sim$ Binomial $(50, 0.5)$. Then $E(X) = 50 \times \frac{5}{10} = 25$ and $\sigma = \sqrt{25 \times 0.5} = 3.54$. We need $P(30 < X < 35) = P(31 \leq X \leq 34) = P(30.5 \leq X \leq 34.5) = P(\frac{30.5-25}{3.54} \leq \frac{X-25}{3.54} \leq \frac{34.5-25}{3.54}) \cong P(1.55 \leq Z \leq 2.68) = \Phi(2.68) - \Phi(1.55) = 0.996319 - 0.939529 = 0.05689$, so we choose B.

- (11) $A$ and $B$ are two events such that $P(A) = 0.3$, $P(B) = 0.5$ and $P(A \cup B) = 0.65$. Are $A$ and $B$ independent events?

  A) NO B) YES C) insufficient information is given

  **Solution:** Note that $P(A)P(B) = 0.15$, and note that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so $P(A \cap B) = 0.8 - 0.65 = 0.15$. Since $P(A)P(B) = P(A \cup B)$ we get independent events, so we choose B.

- (12) Assume that $X$ is an exponential random variable with parameter $\lambda = 10$. Find $P(10 < X < 20 | X > 5)$.

  A) $e^{-25} - e^{-15}$ B) $e^{-100} - e^{-200}$ C) $e^{-40} - e^{-300}$ D) $e^{-50} - e^{-150}$ E) none of the preceding

  **Solution:** We have $P(10 < X < 20 | X > 5) = \frac{P(10<X<20 \text{ and } X>5)}{P(X>5)} = \frac{P(10<X<20)}{P(X>5)} = \frac{P(X>10)-P(X>20)}{P(X>5)} = \frac{e^{-10\times10}-e^{-10\times20}}{e^{-10\times15}} = \frac{e^{-100}-e^{-200}}{e^{-50}} = e^{-50} - e^{-150}$, so we choose D.

- (13) In a factory from planet MathematiX, machines $A$, $B$, $C$ are producing pencils of the same length. Machines $A$, $B$, $C$ produce 2%, 1%, and 3% defective pencils, respectively. Of the total production of pencils, machine $A$ produces 35%, machine $B$ produces 25%, and machine $C$ produces 40%. We select one pencil at random from the total pencils produced in a day. If the pencil is defective, find the probability that it was produced by machine $C$.

  A)$\frac{120}{215}$ B) $\frac{140}{215}$ C) $\frac{160}{215}$ D) $\frac{180}{215}$ E) none of the preceding

  **Solution:** We have $P(D) = P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C) = \frac{35}{100}\frac{2}{100} + \frac{25}{100}\frac{1}{100} + \frac{40}{100}\frac{3}{100} = \frac{215}{10000}$, and $P(C|D) = \frac{P(C)P(D|C)}{P(D)} = \frac{120}{215}$, so we choose A.

- (14) This is a bonus problem. NO partial marks! To get the 2 extra points you have to convince me why your solution is correct!

  The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 seconds. What is the reaction time that is exceeded 90% of the time?

  Solution: We need to find $x$ such that $0.90 = P(X > x) = 1 - P(X \leq x) = 1 - P(\frac{X-0.4}{0.05} \leq \frac{x-0.4}{0.05}) = 1 - P(Z \leq \frac{x-0.4}{0.05})$, where $Z$ is standard normal. So $\Phi(\frac{x-0.4}{0.05}) = 0.10$, and by the tables one may continue as follows: $\frac{x-0.4}{0.05} = -1.28$, so $x = 0.05 \times (-1.28) + 0.4 = 0.336$. **Alternative:** $\frac{x-0.4}{0.05} = -1.285$, hence $x = 0.05 \times (-1.285) + 0.4 = 0.33575$. BOTH WAYS ARE GOOD ENOUGH TO GET THE POINTS!