# MATH 2377, SUMMER 2009
# ASSIGNMENT 6

**Q1; 5 points**. Assume $n = 5$, and consider the following data associated to a simple linear regression model:

| x | y |
|---|---|
| -2 | 0 |
| -1 | 0 |
| 0 | 1 |
| 1 | 1 |
| 2 | 3 |

a) Compute the values $\hat{\beta}_1$ and $\hat{\beta}_0$;

b) If $\sigma^2$ is the variance of the random error, compute the variances of the estimators $\hat{\beta}_1$ and $\hat{\beta}_0$;

c) Compute an estimation for $\sigma$.

**Solution: a)** Note that $n = 5$, $\sum_{i=1}^{5} x_i = 0$, $\sum_{i=1}^{5} y_i = 5$; $\sum_{i=1}^{5} x_i y_i = 7$, $\sum_{i=1}^{5} x_i^2 = 10$, then one may obtain $\bar{x} = 0$ and $\bar{y} = 1$. So we may compute the following point estimates: $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{7-5\times 0\times 1}{10-5\times 0^2} = 7/10 = 0.7$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \times \bar{x} = 1$.

**b)** So we need $Var(\hat{\beta}_0)$ and $Var(\hat{\beta}_1)$. We do have the following computations: $Var(\hat{\beta}_1) = \frac{\sigma^2}{S_{xx}} = \frac{\sigma^2}{10}$ by **a)**. Note that $Var(\hat{\beta}_0) = (\sigma^2) \times \left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right] = (\sigma^2) \times \{\frac{1}{5} + 0\} = \frac{\sigma^2}{5}$.

**c)** Note that $\hat{\sigma}^2 = \frac{SS_E}{n-2} = \frac{SS_E}{3} = \frac{S_{yy} - \hat{\beta}_1 \times S_{xy}}{3} = \frac{(\sum y_i^2 - n\bar{y}^2) - (0.7)\times 7}{3} = \frac{(11-5\times 1^2) - 0.7\times 7}{3} =$ 0.366666666, hence an estimation for $\sigma$ is 0.60553007.

**Marking scheme:** a) 1 point for $\hat{\beta}_1$, and 1 point for $\hat{\beta}_0$; b) 1 point for **each** variance; c) 1 point.

**Q2; 3 points**. Do problem 11-22: **for part a) test only for signifiance**; b) estimate the standard errors of the slope and intercept.

**a)** We have $H_0 : \beta_1 = 0$, $H_1 : \beta_1 \neq 0$. We compute the observed value as follows: $t_0 = \frac{\hat{\beta}_1 - 0}{\sqrt{\hat{\sigma}^2/S_{xx}}} = \frac{0.00416}{\sqrt{\frac{0.00797}{143215.8-(1478)^2/20}}} = \frac{0.00416}{0.000484755} = 8.58165465$, and w eget that $t_{\alpha/2,n-2} = t_{0.025,18} = 2.101$. Since $t_0 > t_{0.025,18}$ we reject $H_0$ and acept $H_1$. We used data/results from lecture on June 23, 2009, page 11.

**b)** Note that $\sqrt{\hat{\sigma}^2/S_{xx}}$ has already appeared above in **a)**, and we get that it equals to $\sqrt{\frac{0.00797}{33916.6}} = 0.000484755$. Note that the other estimated standard error is given by $\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right]} = \sqrt{0.00797\left[\frac{1}{20} + \frac{(1478/20)^2}{33916.6}\right]} \cong 0.04$.

**Marking scheme:** a) 1 point or nothing; b) 1 point for each estimated standard error.

**Q3; 4 points**. Do problem 11-38 on page 415.

Sol: **a,b)** For the slope: the interval is given by $\hat{\beta}_1 \pm t_{\alpha/2,n-2}\hat{\sigma}_{\hat{\beta}_1} = 0.00416 \pm t_{0.005,18} \times \hat{\sigma}_{\hat{\beta}_1} = 0.00416 \pm (2.878) \times (0.000484) = [0.00276, 0.00555]$.

For the intercept: the interval is given by $\hat{\beta}_0 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right]} = 0.32999 \pm (2.878) \times (0.04) = [0.21, 0.44]$.

**c)** Note that $x_0 = 85$, and so $\hat{\mu}_{Y|x_0} = \hat{\beta}_0 + \hat{\beta}_1 \times x_0 = 0.32999 + 0.00416 \times 85 = 0.68359$. So the interval is given by: $\hat{\mu}_{Y|x_0} \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}\right]} = 0.68359 \pm (2.878) \times \sqrt{0.00797\left[\frac{1}{20} + \frac{(85-73.9)^2}{33991.6}\right]} = [0.624, 0.743]$.

**d)** Note that $x_0 = 90$, and that $\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 \times x_0 = 0.32999 + 0.00416 \times 90 = 0.70439$. So the interval is given by: $\hat{y}_0 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[1 + \frac{1}{n} + \frac{(x_0-\bar{x})^2}{S_{xx}}\right]} = 0.70439 \pm (2.878) \times \sqrt{0.00797\left[1 + \frac{1}{20} + \frac{(85-73.9)^2}{33991.6}\right]} = [0.440, 0.968]$.

**Marking scheme: 1 point for each of** a),b)c),d)

**Q4, 3 points**. Do problem **11-64 a) and no *p*-value** on page 425.

Sol: We see that $n = 20$, $\alpha = 0.05$, so $\alpha/2 = 0.025$, moreover $H_0 : \rho = 0$, $H_1 : \rho \neq 0$. The observed value of the test statistics is $t_0 = \frac{0.8 \times \sqrt{n-2}}{\sqrt{1-0.8^2}} = 5.656854$. From the table we get that $t_{\alpha/2,n-2} = t_{0.025,18} = 2.101$. Since $|t_0| > t_{\alpha/2,n-2}$ we reject $H_0$.

**Marking scheme:** 1 point for $t_0$, 1 point for $t_{\alpha/2,n-2}$, 1 point for the right conclusion.