MATH 2377, SUMMER 2009
ASSIGNMENT 5

9-6. **(3 points)** a) So $n = 16$, the critical region is $\bar{x} < 11.5$, $\sigma = 0.5$. Since we are computing α, we take into account H_0 is true, i.e., $\mu = 12$. Hence $\alpha = P(\overline{X} < 11.5, \mu = 12) = P(\frac{\overline{X}-12}{\sigma/\sqrt{n}} < \frac{11.5-12}{\sigma/\sqrt{n}}) = P(Z < -4) = 0$ (by the table!) b) We compute (recall the acceptance region) $\beta = P(\overline{X} \geq 11.5; \mu = 11.25) = P(\frac{\overline{X}-11.25}{\sigma/\sqrt{n}} \geq \frac{11.5-11.25}{\sigma/\sqrt{n}}) = P(Z \geq \frac{0.25}{0.5/\sqrt{16}}) = 1 - P(Z < 2) = 1 - 0.977250 = 0.02275$; c) We compute $\beta = P(\overline{X} \geq 11.5; \mu = 11.5) = P(\frac{\overline{X}-11.5}{\sigma/\sqrt{n}} \geq \frac{11.5-11.5}{\sigma/\sqrt{n}}) = P(Z \geq 0) = 1 - P(Z < 0) = 1 - 0.5 = 0.5$.

Marking scheme: 1 point for a), 1 point for b), 1 point for c)

9-36. **(4 points)** a) $\alpha = 0.05$, $H_0 : \mu = 100$, $H_1 : \mu > 100$, the observed value is $z_0 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ where $\bar{x} = 98$, $\sigma = 2$, and $n = 9$. Hence $z_0 = \frac{98-100}{2/\sqrt{9}} = -3$. Since z_0 is not bigger than $z_{0.05} = 1.645$, we fail to reject H_0, in other words: the temperature is not significantly different greater than 100 at the level $\alpha = 0.05$; b) the p-value is computed (see **page 15 of** our lecture notes) as follows $1 - \Phi(-3) = 1 - 0.001350 = 0.99865$; c) We need to get β (read the statement). **Use page 17 of our lecture notes** (but the 1-sided version!), set $\delta = \mu_1 - \mu_0 = 104 - 100$ and get $\beta = P(\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} - \frac{\delta\sqrt{n}}{\sigma} \leq z_\alpha - \frac{\delta\sqrt{n}}{\sigma}) = P(Z \leq z_\alpha - \frac{\delta\sqrt{n}}{\sigma}) = P(Z \leq 1.645 - \frac{4\times 3}{2}) = P(Z \leq -4.355) = 0$ by the table.

Marking scheme: 1 point for a), 1 point for b), 2 points for c)

9-50. **(1 point)** a) We do have: $H_0 : \mu = 22.5$, $H_1 : \mu \neq 22.5$, $\alpha = 0.05$, $t_0 = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{22.496-22.5}{0.378/\sqrt{5}} = \frac{-0.004}{0.378/\sqrt{5}} = -0.00237$. Since $t_{\alpha/2,n-1} > t_0 = -0.00237 > -t_{0.025,4} = -2.776$ we fail to reject H_0.

Marking scheme: 1 point for a)

9-78. **(1 point)** a) From the statement we get $H_0 : p = 0.05$, $H_1 : p \neq 0.05$, $\alpha = 0.05$, $x = 13$, $n = 300$, $\hat{p} = 13/300 = 0.043$, $z_0 = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} = -0.53$. Since $z_0 = -0.53 > -1.65 = z_{\alpha/2}$ we fail to reject H_0.

Marking scheme: 1 point for a)

5-34. **(2 points)** Note that $E(X) = \sum_x xf_{XY}(x,y) = (-1)\frac{1}{8} + (-0.5)\frac{1}{4} + (0.5)\frac{1}{2} + (1)\frac{1}{8} = 0.125$, $E(Y) = \sum_y yf_{XY}(x,y) = (-2)\frac{1}{8}+(-1)\frac{1}{4}+(1)\frac{1}{2}+(2)\frac{1}{8} = 0.25$, $E(XY) = \sum_{x,y} xyf_{XY}(x,y) = (-1)(-2)\frac{1}{8}+(-0.5)(-1)\frac{1}{4}+(0.5)(1)\frac{1}{2}+(1)(2)\frac{1}{8} = 0.875$; Next note that $E(X^2) = \frac{1}{8}+(0.5)^2\frac{1}{4}+(0.5)^2\frac{1}{2} + \frac{1}{8} = 0.4375$, so $Var(X) = 0.4375 - 0.125^2 \cong 0.4219$, similarly $Var(Y) = 1.6875$. It follows that $\sigma_{XY} = E(XY) - \mu_X\mu_Y = 0.875 - (0.125) \times (0.25) = 0.84375 \cong 0.8438$. We get $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0.8438}{\sqrt{0.4219}\sqrt{1.6875}} = 1$.

Marking scheme: 1 point for ρ_{XY}, 1 point for σ_{XY}

5-18. **(4 points)** From $1 = c\int_0^3\int_x^{x+2}(x+y)dydx = c\int_0^3(xy + \frac{y^2}{2})|_x^{x+2}dx = c\int_0^3\{2x + 2x + 2\}dx = c\int_0^3(4x+2)dx = c\{2x^2 + 2x)|_0^3\} = c \times 24$ we get $c = 1/24$.

a) $P(X < 1, Y < 1) = (\frac{1}{24})\int_0^1\int_x^2(x+y)dydx = 0.104166666 \approx 0.10417$;

b) $P(1 < X < 2) = (\frac{1}{24}) \int_1^2 \int_x^{x+2} (x+y)dydx = 1/3$;
c) $P(Y > 1) = 1 - P(Y \leq 1) = 1 - (\frac{1}{24}) \int_0^1 \int_x^1 (x+y)dydx = 47/48 \cong 0.979166666$;
d) $P(X < 2, Y < 2) = (\frac{1}{24}) \int_0^2 \int_x^2 (x+y)dydx = 1/6$;
e) $E(X) = (\frac{1}{24}) \int_0^3 \int_x^{x+2} x(x+y)dydx = (\frac{1}{24}) \int_0^3 (4x^2 + 2x)dx = 15/8$;
f) $Var(X) = (\frac{1}{24}) \int_0^3 \int_x^{x+2} x^2(x+y)dydx - (\frac{15}{8})^2 = (\frac{1}{24}) \int_0^3 (4x^3 + 2x^2)dx - (\frac{15}{8})^2 = 39/64$;
g) $f_X(x) = (\frac{1}{24}) \int_x^{x+2} (x+y)dy = \frac{1}{6}x + \frac{1}{12}$ for $0 < x < 3$.

Marking scheme: 1 point for getting c, 1 point for each of a)-g)