MATH 2377, SUMMER 2009
ASSIGNMENT 4

7-14. **(2 points)** Note that (using the properties of expectation) $E(\overline{X}_1) = E(\frac{1}{2n}\sum_{i=1}^{2n} X_i) = \frac{1}{2n}E(\sum_{i=1}^{2n} X_i) = \frac{1}{2n}\{\mu + \cdots + \mu\} = \frac{2n\mu}{2n} = \mu$ and $E(\overline{X}_2) = E(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n}E(\sum_{i=1}^{n} X_i) = \frac{1}{n}\{\mu + \cdots + \mu\} = \frac{n\mu}{n} = \mu$, so both ($\overline{X}_1$ and $\overline{X}_2$) are unbiased estimators of μ. Using the properties of variance one gets that the variances are $Var(\overline{X}_1) = \frac{1}{4n^2}Var(X_1) + \cdots + \frac{1}{4n^2}Var(X_{2n}) = \frac{2n\sigma^2}{4n^2} = \frac{\sigma^2}{2n}$ and $Var(\overline{X}_2) = \frac{1}{n^2}Var(X_1) + \cdots + \frac{1}{n^2}Var(X_n) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$. Since $\frac{\sigma^2}{2n} \le \frac{\sigma^2}{n}$ we conclude that $\overline{X}_1$ is the better estimator.
Marking scheme: 1 pt for Es and Vars; 1 point for finding the better estimator
8-10. **(2 points)** We know from the statement that $\sigma = 0.01$; $\overline{x} = 1.5045$, $n = 10$, $\alpha = 0.01$, so $\alpha/2 = 0.005$ (since it is 2-sided!), hence $z_{\alpha/2} = 2.58$. So the interval is $\overline{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \overline{x} + z_{\alpha/2}\sigma/\sqrt{n}$, or $1.5045 - 2.58 \times 0.01/\sqrt{10} \le \mu \le 1.5045 + 2.58 \times 0.01/\sqrt{10}$ or $1.4963 \le \mu \le 1.5127$.
Marking scheme: all or nothing
8-16. **(2 points)** From both statements we get $E = 15$; $\alpha = 0.01$, so $\alpha/2 = 0.005$, hence $z_{\alpha/2} = 2.58$. Moreover $\sigma^2 = 1000$, so $\sigma = 31.62$. Then we require $n = (\frac{z_{\alpha/2}\sigma}{E})^2 = 29.6$. We round up and we require $n = 30$.
Marking scheme: all or nothing
8-36. **(2 points)** From the statement we get that $n = 25$; $\overline{x} = 4.05$, $s = 0.08$, $\alpha = 0.05$. The interval is $\overline{x} - t_{\alpha,n-1}s/\sqrt{n} \le \mu$. Since by table V one gets that $t_{\alpha,n-1} = t_{0.05,24} = 1.711$, our interval is in fact $4.05 - 1.711 \times \{0.08/\sqrt{25}\} \le \mu$, or $4.023 \le \mu$. We can say with a high confidence that the true value of the mean is greater than 4.023.
Marking scheme: all or nothing
8-52. **(3 points)** a) The interval is $[\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$, where $n = 50$, $X = 18$, $\hat{p} = 18/50 = 0.36$, $\alpha = 0.05$, $\alpha/2 = 0.025$, $z_{\alpha/2} = 1.96$. We obtain $0.36 - 1.96\sqrt{\frac{0.36\times0.64}{50}} \le p \le 0.36 + 1.96\sqrt{\frac{0.36\times0.64}{50}}$, or $0.227 \le p \le 0.493$; b) Using $n = (\frac{z_{\alpha/2}}{E})^2 p(1-p)$ where $p = 0.36$ and $E = 0.02$ one gets $n = (\frac{1.96}{0.02})^2(0.36)(0.64) = 2212.76$. Rounding up we get $n = 2213$; c) Here we use the formula $n = (\frac{z_{\alpha/2}}{E})^2(0.25) = 2401$.
Marking scheme: 1 point for a), 1 point for b), 1 point for c)
8-46. **(4 points)** a) The interval we are looking for is $[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}]$, where $n = 10$, $\alpha = 0.01$, so $\alpha/2 = 0.005$. We compute $s = 1.913$. Thus by table IV one gets $\frac{9\times(1.913)^2}{23.59} \le \sigma^2 \le \frac{9\times(1.913)^2}{1.73}$, or $1.396 \le \sigma^2 \le 19.038$; b) The lower bound for σ^2 is $\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}}$. Hence $\frac{9\times(1.913)^2}{21.67} \le \sigma^2$, or $1.5199 \le \sigma^2$; c) Now $\alpha = 0.10$, $n = 10$, so $\chi^2_{0.10,9} = 14.68$. So we get $\frac{9\times(1.913)^2}{14.68} \le \sigma^2$, or $1.498 \le \sigma$; d) I) The lower confidence bound of the 99% two-sided interval is less than the one-sided interval; II) the lower confidence bound for the variance (σ^2) in

part (c) is greater because the confidence is lower.
Marking scheme: 1 point for a), 1 point for b), 1 point for c), 1 point for d)