MATH 2377, SUMMER 2009
ASSIGNMENT 3

4-54. **(1.5 points)** a) Note that $X \sim N(12.4, (0.1)^2)$ where X is the rv fill volume! We compute $P(X < 12) = P(\frac{X-12.4}{0.1} < \frac{12-12.4}{0.1}) = P(Z < -4)$, where Z is now standard normal, and so $P(Z < -4) = \Phi(-4) = 0$ by the tables. b) We need $P(X < 12.1 \text{ or } X > 12.6) = P(X < 12.1) + P(X > 12.6)$ (since the events are m.exclusive!) $= P(\frac{X-12.4}{0.1} < \frac{12.1-12.4}{0.1}) + P(\frac{X-12.4}{0.1} > \frac{12.6-12.4}{0.1}) = P(Z < -3) + P(Z > 2) = 0.001350 + 1 - P(Z \leq 2) = 0.001350 + 1 - 0.977250 = 0.0241$ or 2.41%; c) The symmetric specifications about the mean = 12.4 are $12.4 - x$ and $12.4 + x$ (of course 12.4 is the midpoint!). We have $0.99 = P(12.4 - x < X < 12.4 + x) = P(\frac{12.4-x-12.4}{0.1}) < \frac{X-12.4}{0.1} < \frac{12.4+x-12.4}{0.1}) = P(\frac{-x}{0.1} < Z < \frac{x}{0.1})$, where Z is standard normal, $= \Phi(\frac{x}{0.1}) - \Phi(\frac{-x}{0.1}) = 2\Phi(\frac{x}{0.1}) - 1$ (see page 20 from the lecture on normal distribution). So $\Phi(\frac{x}{0.1}) = \frac{1+0.99}{2}$. By tables $\frac{x}{0.1} = 2.575$, so $x = (0.1)2.575 = 0.2575$ or if you want to round: $\cong 0.258$.

Marking scheme: 0.5 point for a), 0.5 point for b), 0.5 point for c)

4-62. **(1.5 points)** From the statement we get $X \sim N(0.002, (0.0004)^2)$ where X is the diameter! a) $P(X > 0.0026) = P(\frac{X-0.002}{0.0004} > \frac{0.0026-0.002}{0.0004}) = P(Z > 1.5) = 1 - \Phi(1.5) = 1 - 0.933193 = 0.066807$ (if you want to round you just get$\cong$ 0.06681); b) $P(0.0014 < X < 0.0026) = P(\frac{0.0014-0.002}{0.0004}) < \frac{X-0.002}{0.0004} < \frac{0.0026-0.002}{0.0004}) = P(-1.5 < Z < 1.5) = \Phi(1.5) - \Phi(-1.5) = 2\Phi(1.5) - 1 = 2(0.933193) - 1 = 0.866386$; c) We need to find σ such that $0.995 = P(\frac{0.0014-0.002}{\sigma}) < Z < \frac{0.0026-0.002}{\sigma})$, so $0.995 = P(\frac{-0.0006}{\sigma}) < Z < \frac{0.0006}{\sigma}) = \Phi(\frac{0.0006}{\sigma}) - \Phi(-\frac{0.0006}{\sigma}) = 2\Phi(\frac{0.0006}{\sigma}) - 1$, hence $\Phi(\frac{0.0006}{\sigma}) = \frac{1.995}{2} = 0.9975$. Using the tables we get $\frac{0.0006}{\sigma} = 2.805$, so $\sigma = 0.0006/2.805 = 0.000213903$ (if you want to round you get $\cong 0.000214$).

Marking scheme: 0.5 point for a), 0.5 point for b), 0.5 point for c)

4-72. **(4 points)** Set X to be the rv "the no. of erros on site". From the statement we get that $X \sim$ Binomial(100,50/1000). We need $P(X \geq 1)$. Recall that $E(X) = np = 100 \times 0.05 = 5$, and $Var(X) = np(1-p) = 5 \times 0.95 = 4.75$. We approximate as follows: $P(X \geq 1) = P(X \geq 1 - 0.5) \cong P(Z \geq \frac{1-0.5-np}{\sqrt{np(1-p)}}) = P(Z \geq \frac{1-0.5-5}{\sqrt{4.75}}) = P(Z \geq -2.06) = 1 - P(Z < -2.06)$ (where Z is standard normal) $= 1 - 0.019699 = 0.980301 (\cong 0.9803)$.

Marking scheme: Binomial(,), expectation and variance 2 points; the approximation 2 points.

4-70. **(2 points)** a) If X is the rv "the no. of accounts that have an error", then $X \sim$ Binomial(0.001,362000). $E(X) = np = 362$, and $Var(X) = np(1-p) = 362 \times 0.999$, so the standard deviation is $19.0167820 \approx 19.0168$; b) $P(X < 350) = P(X \leq 349) \cong P(Z \leq \frac{349.5-362}{19.0168}) = P(Z \leq -0.66) = 0.254627 \cong 0.255$; c) We need x such that $P(X > x) = 0.05$. We have $1 - P(X \leq x) = 0.05$, so $P(X \leq x) = 0.95$. Using the approximation one gets $0.95 = P(X \leq x + 0.5) \cong \Phi(\frac{x+0.5-np}{\sqrt{np(1-p)}})$. We obtain $1.645 = \frac{x+0.5-362}{19.0168}$, and so $x = (1.645) \times (19.0168) + (362 - 0.5) \approx 392.7$; d) We need (since we are dealing with 2 months) $P(X > 400 \text{ and } X > 400) = P(X > 400)^2$ because of they are independent events (as stated in the problem). We do have $P(X > 400) = 1 - P(X \leq 400) = 1 - P(X \leq 400.5) \approx 1 -$

$P(Z \le \frac{400.5-362}{19.0168}) = 1 - \Phi(2.0245) = 1 - \frac{0.978308+0.978822}{2} = 1 - 0.978565 = 0.021435 \cong 0.0215$.
So the sought probability is $0.0215^2 = 0.00046225$.
Marking scheme: 0.5 point for a), 0.5 point for b), 0.5 point for c), 0.5 point for d)
4-86. **(2 points)** a) For Poisson: rate = mean; since we are dealing with exponential here, we get that the expectation is $\frac{1}{2.3} = 0.4348$; b) 3 months means 0.25 years; so the new λ is $\frac{2.3\times(1/4)}{1} = 0.5750$. We compute $P(X = 0)$ where X is the rv "the no. of sightings". So $P(X = 0) = \frac{e^{-0.5750}\times(0.5750)^0}{0!} = 1/1.777130527 = 0.5627$; c) Set Y be the rv "time between sightings", then Y us exponential with rate 2.3. We compute: $P(Y > 0.5) = e^{-2.3\times0.5} \cong 0.316636$; d) The new λ is $\frac{3\times2.3}{1} = 6.9$, and we compute (see the def of X) the following probability $P(X = 0)$ as follows: $P(X = 0) = \frac{e^{-6.9}\times(6.9)^0}{0!} = 0.001007 \cong 0.001$.
Marking scheme: 0.5 point for a), 0.5 point for b), 0.5 point for c), 0.5 point for d)
4-102. **(4 points)** We see that $\lambda = 20$; a) When dealing with a Gamma distribution, we know the expectation (read mean) is $\frac{r}{\lambda}$, so we get $\frac{100}{20} = 5$ minutes; b) The difference is $\frac{5\times80}{100} - \frac{5\times50}{100} = 1.5$ minutes; c) Set X be the rv "the no. of calls before 15 secs". Then X is Poisson, and the λ is $\frac{20\times15}{60} = 5$. We need to compute $P(X \ge 3)$ as follows: $P(X \ge 3) = 1 - P(X \le 2) = 1 - e^{-5}\{\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!}\} = 0.8753$

Marking scheme: 1 point for a), 1 point for b), 2 points for c)