MATH 2377, SUMMER 2009
ASSIGNMENT 2

3-72. **(2 points)** a) Set Y to be the rv : the # of questions that are answered corectly! From the statement we learn that $Y \sim B(25, 0.25)$, (since there are 4 answers). We need to compute $P(Y \geq 20) = P(Y = 20) + P(Y = 21) + P(Y = 22) + P(Y = 23) + P(Y = 24) + P(Y = 25) = \binom{25}{20}(0.25)^{20}(0.75)^5 + \binom{25}{21}(0.25)^{21}(0.75)^4 + \binom{25}{22}(0.25)^{22}(0.75)^3 + \binom{25}{23}(0.25)^{23}(0.75)^2 + \binom{25}{24}(0.25)^{24}(0.75)^1 + \binom{25}{25}(0.25)^{25}(0.75)^0 = \frac{9.677}{10^{10}}$.

For part b) we need to compute $P(Y < 5) = P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = \binom{25}{0}(0.25)^0(0.75)^{25} + \binom{25}{1}(0.25)^1(0.75)^{24} + \binom{25}{2}(0.25)^2(0.75)^{23} + \binom{25}{3}(0.25)^3(0.75)^{22} + \binom{25}{4}(0.25)^4(0.75)^{21} = 0.2137$

Marking scheme: 1 point for a) and 1 point for b).

3-110. **(4 points)** a) Set Y to be the rv : the # of calls in 1 hour! From the statement we learn that $Y \sim$ Poisson with a rate $\lambda = 10$; We need to compute $P(Y = 5) = \frac{e^{-10} \times 10^5}{5!} = 0.0378$;

b) We have $P(Y \leq 3) = P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) = e^{-10}\{1 + \frac{10}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!}\} = 0.0103$;

c) For 2 hours the rate is 20, so we need to compute $P(X = 15)$, where X is the rv : the # of calls in 2 hours! Hence $P(X = 15) = e^{-20} \times \frac{20^{15}}{15!} = 0.0516$;

d) For 30 minutes the rate is 5, so we need to compute $P(Z = 5)$, where Z is the rv : the # of calls in 30 minutes! Hence $P(Z = 5) = e^{-5} \times \frac{5^5}{5!} = 0.1755$.

Marking scheme: 1 point for a), 1 point for b), 1 point for c), 1 point for d).

3-16. **(4 points)** Since $f(x) \geq 0$ for all $x = 1, 2, 3$, and $f(1) + f(2) + f(3) = \frac{8}{7} \times \{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\} = \frac{8}{7} \times \frac{7}{8} = 1$ we get that f is indeed a p.m.f.

a) $P(X \leq 1) = P(X = 1) = f(1) = 0.5714$;

b) $P(X > 1) = P(X = 2) + P(X = 3) = f(2) + f(3) = 0.4286$;

c) $P(2 < X < 6) = P(X = 3) = f(3) = 1/7 = 0.1429$;

d) $(X \leq 1 \text{ or } X > 1) = 1$, since that event is the whole sample space.

Marking scheme: 1 point for a), 1 point for b), 1 point for c), 1 point for d).

4-2. **(1 point)** a) $0.10 = P(x < X) = \int_x^\infty f(z)dz = \int_x^\infty e^{-z}dz = -e^{-z}|_x^\infty = e^{-x}$, so $\ln(0.10) = -x$; hence $x = -\ln(0.10) = 2.3$;

b) $0.10 = P(X \leq x) = \int_0^x f(z)dz = \int_0^x e^{-z}dz = -e^{-z}|_0^x = 1 - e^{-x}$, so $e^{-x} = 0.90$, $-x = \ln(0.90)$; $x = -\ln(0.90) = 0.1054$;

Marking scheme: 0.5 point for a), 0.5 point for b).

4-8. **(3 points)** a) $P(X < 74.8) = \int_{74.6}^{74.8} f(z)dz = \int_{74.6}^{74.8} 1.25dz = 1.25(74.8 - 74.6) = 1.25 \times 0.2 = 0.25$;

b) $P(X < 74.8 \text{ or } X > 75.2) = P(X < 74.8) + P(X > 75.2)$ since we are dealing with mutually exclusive events! Hence $P(X < 74.8) + P(X > 75.2) = 0.25 + 0.25 = 0.5$ (use a) for the first integral, the last integral is computed as in a))!

c) We need to compute $P(74.7 < X < 75.3) = \int_{74.7}^{75.3} f(z)dz = \int_{74.7}^{75.3} 1.25dz = (75.3 - 74.7) \times 1.25 = 0.6 \times 1.25 = 0.750$.

Marking scheme: 1 point for a), 1 point for b), 1 point for c)

4-28. **(1 point)** a) Let X be the cable length; $E(X) = \int_{1200}^{1210} xf(x)dx = \int_{1200}^{1210} x(0.1)dx = (0.1) \times \frac{x^2}{2}|_{1200}^{1210} = 1205$; $Var(X) = \int_{1200}^{1210}(x - 1205)^2 f(x)dx = \int_{1200}^{1210}(x - 1205)^2(0.1)dx = (0.1)\frac{(x-1205)^3}{3}|_{1200}^{1210} = 8.333$, so the standard dev is $\sqrt{8.333} = 2.877$.

b) $P(1195 < X < 1205) = P(1200 < X < 1205) = \int_{1200}^{1205} f(x)dx = \int_{1200}^{1205} 0.1dx = (0.1)(1205 - 1200) = 0.5$.

Marking scheme: 0.5 point for a), 0.5 point for b)