MATH 2377, SUMMER 2009
ASSIGNMENT 1

2-24. **(2 points)**

The sample space is $S = \{l | l = 0, 1, 2, \dots\}$ and:

a) $A = \{l \in S | 675 \leq l \leq 700\} = \{l \in S | l = 675, 676, \dots, 700\}$ (the unit is nm);

b) $B = \{l \in S | 450 \leq l \leq 500\}$, or $B = \{l \in S | l = 450, 451, \dots, 500\}$;

c) $A \cap B = \emptyset$ - the empty set since $500 < 675$;

d) $A \cup B = \{l \in S | 450 \leq l \leq 500 \text{ or } 675 \leq l \leq 700\}$, or $A \cup B = \{l \in S | l = 450, 451, \dots, 500, 675, 676, \dots, 700\}$.

Marking scheme: 0.5 for each of a), b), c), d);

2-62. **(3 points)**

a) $P(A) = \frac{86}{100}$;

b) $P(B) = \frac{79}{100}$;

c) $P(A^{'}) = \frac{14}{100}$;

d) $P(A \cap B) = \frac{70}{100}$;

e) $P(A \cup B) = \frac{95}{100}$;

f) $P(A^{'} \cup B) = \frac{84}{100}$;

Marking scheme: 0.5 for each of a), b), c), d), e), f);

2-66. **(3 points)**

a) $P(A^{'}) = 1 - P(A) = 1 - 0.3 = 0.7$;

b) Using the formula given in class we get: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.2 - 0.1 = 0.4$;

c) Since $B = (B \cap A) \cup (B \cap A^{'})$, we get $P(B) = P(B \cap A) + P(B \cap A^{'})$, hence we obtain: $P(B \cap A^{'}) = P(B) - P(B \cap A) = 0.2 - 0.1 = 0.1$;

d) Since $A = (A \cap B) \cup (A \cap B^{'})$, we get $P(A) = P(A \cap B) + P(A \cap B^{'})$, hence we obtain: $P(A \cap B^{'}) = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$;

e) $P[(A \cup B)^{'}] = 1 - P(A \cup B) = 1 - 0.4 = 0.6$, (see b));

f) $P(A^{'} \cup B) = P(A^{'}) + P(B) - P(B \cap A^{'}) = 0.7 + 0.2 - 0.1 = 0.8$, see c).

Marking scheme: 0.5 for each of a), b), c), d), e), f);

2-78. **(3 points)**

a) $P(A) = \frac{82}{100}$;

b) $P(B) = \frac{90}{100}$;

c) $P(A|B) = \frac{A \cap B}{P(B)} = \frac{80}{90}$;

d) $P(B|A) = \frac{B \cap A}{P(A)} = \frac{80}{82}$;

e) the probability we are looking for is just $P(B|A) = 0.9756$;

f) the probability we are looking for is just $P(A|B^{'}) = \frac{P(A \cap B^{'})}{P(B^{'})} = 0.02/(1 - 0.9) = 2/10 = 0.2$

Marking scheme: 0.5 for each of a), b), c), d), e), f);

2-92. **(2 points)**

I Denote by **FLAW** the event: a roll contains a flaw; Denote by **COTTON** the event: a roll is cotton; Then $P(\mathbf{COTTON}) = 0.7$, and so $P((\mathbf{COTTON})') = 0.3$. From the text we get that: $P(\mathbf{FLAW}|\mathbf{COTTON}) = 0.02$ and $P(\mathbf{FLAW}|(\mathbf{COTTON})') = 0.03$.

II By the rule given in class we get: $P(\mathbf{FLAW}) = P(\mathbf{FLAW}|\mathbf{COTTON})P(\mathbf{COTTON}) + P(\mathbf{FLAW}|(\mathbf{COTTON})')P((\mathbf{COTTON})') = 0.02 \times 0.7 + 0.03 \times 0.3 = 0.023$.

Marking scheme: 1 point for each *I* and *II*.

2-122. **(2 points)**

Let D be the event: a selected item is defective. Let A be the event: the inspector classify an item as defective. From the statement we have: $P(D) = 0.009$, and $P(A|D) = 0.99$ and $P(A|D') = 0.005$.

a) We need to find $P(A)$. We have $P(A) = P(A|D)P(D) + P(A|D')P(D') = 0.99 \times 0.009 + 0.005 \times 0.991 = 0.013865$

b) We need to find $P(D'|A')$. We have $P(D'|A') = \frac{P(D' \cap A')}{P(A')} = \frac{P(A'|D')P(D')}{1-P(A)} = \frac{[1-P(A|D')][1-P(D)]}{1-P(A)} = \frac{[1-0.005][1-0.009]}{1-0.013865} = 0.99991$.

Marking scheme: 1 point for each a) and b).