ARE YOU READY? ☺

13/210 Since A has size $m \times n \Rightarrow \operatorname{rank} A \le m$, $\boxed{\operatorname{rank} A \le n}$ (from a Theorem in class). But $\operatorname{rank} A =$
$= m \Rightarrow m \le n$. Done.

15/210 a) Let v be in $\operatorname{col} B$. I want to show that v is in $\operatorname{Null}(A)$:

Since v is in $\operatorname{Col} B \Rightarrow v = BX$ for some X in $\mathbb{R}^k$.

NOTE that: $Av = A(BX) = (AB)X = 0X = 0$.

$\boxed{\text{SO}}$: v is in $\operatorname{Null}(A)$.

b) IDEA: Let $B = [b_1\ b_2\ \dots\ b_k]$ where b_i is the i^{th} Column of B. Now: each b_i is in $\operatorname{Col} B$!! So b_i is in $\operatorname{Null}(A)$ - (why? Read b) again!) - hence $Ab_i = 0$.

Recall: $AB = A[b_1 b_2 \cdots b_k] = [Ab_1\ Ab_2\ \cdots\ Ab_k]$
$= [0\ 0 - 0] = 0$. Done

$\boxed{17/202}$ a) Suppose $t_1(aX + bY) + t_2(cX + dY) = 0$.

Then: $(t_1 a + t_2 c)X + (t_1 b + t_2 d)Y = 0$. Since $\{X, Y\}$ is L.I. (recall the statement!) $\Rightarrow \begin{cases} t_1 a + t_2 c = 0 \\ t_1 b + t_2 d = 0 \end{cases} \Rightarrow$

$\left[\begin{array}{cc|c} a & c & 0 \\ b & d & 0 \end{array}\right]$, $\underbrace{\begin{matrix} a & c \\ b & d \end{matrix}}_{A}$. Since A is invertible $\Rightarrow$ unique solution: $t_1 = 0$, $t_2 = 0$.

So: $\{aX + bY, cX + dY\}$ is L.I.

Since $\dim \mathbb{R}^2 = \boxed{2}$ and $\{aX+bY, \ cX+dY\}$ is L.i.

click

$\boxed{2}$ elements

it follows: $\{aX+bY, \ cX+dY\}$ is A BASIS!

TRY yourself b) !!!

10/c/202: (F) $\{X, \ X+Y, \ Y\}$ is NOT L.i., So

it can $\boxed{\text{NOT}}$ be a basis!!

why is not L.i.? look at: $(-1)X + (1)(X+Y) + (-1)Y$

$= 0$!!! $\underline{(1 \neq 0)}$

20/202 (1) IF $U = \{0\}$ we are Done.

(2) IF $U \neq \{0\}$. Let u be in U, $u \neq 0$.

NOTE That $\{u\}$ is L.I. So it can be enlarged to a basis of U. So (!) $1 \leq \dim U \leq \dim W = 1$.

It follows: $\rightarrow$ $\left.\begin{array}{l} \underline{\dim U = \dim W = 1.} \\ \underline{\text{But}} \ \ U \subseteq W \end{array}\right\} \Rightarrow \boxed{U = W}$

by a thm in class

TIRED??? ?!

13 c/81 CASE 1°) $x \neq 0$; $A \xrightarrow{R_2 - xR_1, \ R_3 - x^2R_1, \ R_4 - x^3R_1}$

$$B = \begin{pmatrix} 1 & x & x^2 & x^3 \\ 0 & 0 & 0 & 1-x^4 \\ 0 & 0 & 1-x^4 & x-x^5 \\ 0 & 1-x^4 & x-x^5 & x^2-x^6 \end{pmatrix} \Rightarrow \det A = \det B =$$

$$= 1 \cdot (-1)^{1+1} \cdot \det \begin{pmatrix} 0 & 0 & 1-x^4 \\ 0 & 1-x^4 & x-x^5 \\ 1-x^4 & x-x^5 & x^2-x^6 \end{pmatrix} =$$

$$= (1-x^4)\,(-1)^{1+3} \det \begin{pmatrix} 0 & 1-x^4 \\ 1-x^4 & x-x^5 \end{pmatrix}$$

$$= (1-x^4)(-1)(1-x^4)^2 = -(1-x^4)^3.$$

CASE 2°: $x=0 \Rightarrow A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \rightarrow \det A = -1.$

! $\boxed{\text{Sol}}$: in general $\det A = -(1-x^4)^3.$

when is $\det A = 0$?
A: $x = \pm 1$

14/81 $\det \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \cos^2\theta + \sin^2\theta =$

$= \underline{1}$ (so the mx. $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is invertible regardless the value of θ).

11/81 $A \xrightarrow{5 \cdot R_1} B \Rightarrow \det B = 5 \cdot \det A$

since $\det B = 10 \Rightarrow \det A = \frac{10}{5} = 2$

GOOD LUCK !!