MAT 1339 A Fall 2010 November 3rd Instructor Catalin Rada

TEST #2

Max = 20

Student Number: ____

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

1. [3 points] Find the derivative of the function $f(x) = \tan(3x) + \cos(2x) - \sin(x)$. Hint: recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Solution: $f(x) = \frac{\sin(3x)}{\cos(3x)} + \cos(2x) - \sin(x)$.

$$\begin{aligned} f'(x) &= \frac{3\cos^2(3x) + 3\sin^2(3x)}{\cos^2(3x)} - 2\sin(2x) - \cos(x) \\ &= \frac{3}{\cos^2(3x)} - 2\sin(2x) - \cos(x) \quad \text{, as } \cos^2(3x) + \sin^2(3x) = 1. \end{aligned}$$

2. [3 points] Find the derivative of $h(x) = \frac{e^{7x}}{\cos(7x)}$.

Solution:

$$h'(x) = \frac{7e^{7x}\cos(7x) + 7e^{7x}\sin(7x)}{\cos^2(7x)}.$$

3. [2 points] Find the equation of the tangent line to the graph of $f(x) = \sin(\sin(x)) + 1$ at $(0, 1) = (0, f(0))$.

Solution:

$f'(x) = \cos\big(\sin(x)\big) \cdot \cos(x)$.
The slope of the tangent line is $f'(0) = \cos\big(\sin(0)\big) \cdot \cos(0) = 1$.
The equation of tangent line is $y - 1 = x$ or $y = x + 1$.

4. [7 points] (i) Solve the equation $e^{7x+2} = 2$, i.e., you must find x;

(ii) Solve the equation $\ln(4x) = 3$, i.e., you must find x;

(iii) Find $f'(3)$ if $f(x) = \cos(\frac{\pi}{2}e^{x-3})$.

Solution:

(i)

$$7x + 2 = \ln 2. \text{ Hence, } x = \frac{\ln 2 - 2}{7}.$$

(ii)

$$4x = e^3. \text{ Hence, } x = \frac{e^3}{4}.$$

(iii)

$$f'(x) = -\sin\left(\frac{\pi}{2} \cdot e^{x-3}\right)\left(\frac{\pi}{2} \cdot e^{x-3}\right)$$

$$\begin{aligned} f'(3) &= -\sin(\frac{\pi}{2}) \cdot (\frac{\pi}{2}) \\ &= -\frac{\pi}{2}. \end{aligned}$$

5. [5 points] Find (if any) the inflection point(s) of $f(x) = e^x(x^2+1)$. When is f concave up? When is f concave down? Hint: construct a table including x, $f(x)$, $f''(x)$.

Solution:

$$f'(x) = e^x(x^2+1) + 2xe^x.$$

$$\begin{aligned} f''(x) &= e^x(x^2+1) + 2xe^x + 2e^x + 2xe^x \\ &= e^x(x^2+1+2x+2+2x) \\ &= e^x(x+1)(x+3) \end{aligned}$$

As $e^x > 0$, $f''(x) = 0$ if and only if $x = -1, -3$.

Interval	$(-\infty,-3)$	$(-3,-1)$	$(-1,\infty)$
Sign of $f''(x)$	$+$	$-$	$+$

Inflection points : $x = -1$ and $x = -3$.
Concave up : $(-\infty,-3) \cup (-1,\infty)$
Concave down : $(-3,-1)$

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• Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

1. [3 points] Find the derivative of the function $f(x) = \tan(4x) + \cos(3x) + \sin(x)$. Hint: recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Solution: $f(x) = \frac{\sin(4x)}{\cos(4x)} + \cos(3x) + \sin(x)$.

$$\begin{aligned} f'(x) &= \frac{4\cos^2(4x) + 4\sin^2(4x)}{\cos^2(4x)} - 3\sin(3x) + \cos(x) \\ &= \frac{4}{\cos^2(4x)} - 3\sin(3x) + \cos(x) \quad \text{, as } \cos^2(4x) + \sin^2(4x) = 1. \end{aligned}$$

2. [3 points] Find the derivative of $h(x) = \frac{e^{7x}}{\sin(7x)}$.

Solution:

$$h'(x) = \frac{7e^{7x}\sin(7x) - 7e^{7x}\cos(7x)}{\sin^2(7x)}.$$

3. [2 points] Find the equation of the tangent line to the graph of $f(x) = \cos(\sin(x)) + x$ at $(0, 1) = (0, f(0))$.

Solution:

$f'(x) = -\sin\big(\sin(x)\big) \cdot \cos(x) + 1.$
The slope of the tangent line is $f'(0) = \sin\big(\sin(0)\big) \cdot \cos(0) + 1 = 1.$
The equation of tangent line is $y - 1 = x$ or $y = x + 1$.

4. [7 points] (i) Solve the equation $e^{7x+1} = 4$, i.e., you must find x;

(ii) Solve the equation $\ln(3x) = 2$, i.e., you must find x;

(iii) Find $f'(2)$ if $f(x) = \cos(\frac{\pi}{2}e^{x-2})$.

Solution:

(i)

$$7x + 1 = \ln 4. \text{ Hence, } x = \frac{\ln 4 - 1}{7}.$$

(ii)

$$3x = e^2. \text{ Hence, } x = \frac{e^2}{3}.$$

(iii)

$$f'(x) = -\sin\left(\frac{\pi}{2} \cdot e^{x-2}\right)\left(\frac{\pi}{2} \cdot e^{x-2}\right)$$

$$\begin{aligned} f'(2) &= -\sin(\frac{\pi}{2}) \cdot (\frac{\pi}{2}) \\ &= -\frac{\pi}{2}. \end{aligned}$$

5. [5 points] Find (if any) the inflection point(s) of $f(x) = e^x(x^2 - 1)$. When is f concave up? When is f concave down? Hint: construct a table including x, $f(x)$, $f''(x)$.

Solution:

$$f'(x) = e^x(x^2 - 1) + 2xe^x.$$

$$\begin{aligned} f''(x) &= e^x(x^2 - 1) + 2xe^x + 2e^x + 2xe^x \\ &= e^x(x^2 - 1 + 2x + 2 + 2x) \\ &= e^x(x^2 + 4x + 1) \\ &= e^x\big(x - (-2 + \sqrt{3})\big)\big(x - (-2 - \sqrt{3})\big) \end{aligned}$$

As $e^x > 0$, $f''(x) = 0$ if and only if $x = -2 + \sqrt{3}, -2 - \sqrt{3}$.

Interval	$(-\infty, -2 - \sqrt{3})$	$(-2 - \sqrt{3}, -2 + \sqrt{3})$	$(-2 + \sqrt{3}, \infty)$
Sign of $f''(x)$	$+$	$-$	$+$

Inflection points : $x = -2 - \sqrt{3}$ and $x = -2 + \sqrt{3}$.
Concave up : $(-\infty, -2 - \sqrt{3}) \cup (-2 + \sqrt{3}, \infty)$
Concave down : $(-2 - \sqrt{3}, -2 + \sqrt{3})$