Important identities: $x^2 - y^2 = (x-y)(x+y)$; $(x+y)^2 = x^2 + 2xy + y^2$; $(x-y)^2 = x^2 - 2xy + y^2$; $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Exercise 1. Find the equation of the tangent line to the graph of $f(x) = 3x^2 - x^3$ at the point $(1, 2)$.

Solution: The equation is $y = mx + n$, and we need to find m (the slope) and n (the y-intercept).

Recall that $m = f'(1)$, so we need $f'(x)$. Now use Power rule, sum rule and get that $f'(x) = 6x - 3x^2$. Hence $f'(1) = 6 - 3 = 3$. It follows that $y = 3x + n$. Since the point $(1, 2)$ belongs to both the graph and **tangent line**, one gets that $2 = 3 + n$, thus $n = 2 - 3 = -1$. We got the equation: $y = 3x - 1$.

Exercise 2. Find the derivatives of the functions: (a) $f(x) = \{2010x^{2010} + 3\}^{77}$;

(b) $g(x) = \frac{x^3 - x}{x^2 + 1}$;

(c) $h(x) = (1 - x + x^5)(2x^3 + x)$.

State what rules you used!

Hints: (a) By Chain rule and Power rule: $(2010x^{2010} + 3)^{77-1}(2010 \times 2010x^{2010-1} + 0) = \dots$ Please finish the computations!

(b) By Quotient rule: $\frac{(3x^2-1)(x^2+1)-(x^3-x)(2x)}{(x^2+1)^2} = \dots$ Please finish the computations!

(c) By Product rule: $(0 - 1 + 5x^4)(2x^3 + x) + (1 - x + x^5)(6x^2 + 1) = \dots$ Please finish the computations!

Exercise 3. Find the derivative of $\sqrt[7]{34x^3 - x^2 + 23}$.

Solution: Our function is in fact $(34x^3 - x^2 + 23)^{\frac{1}{7}}$, thus by chain rule and power rule one gets $\frac{1}{7}(34x^3 - x^2 + 23)^{\frac{-6}{7}}(102x^2 - 2x + 0)$.

Exercise 4. Find the vertical and horizontal asymptotes (if any) of $k(x) = \frac{x-9}{x^2-6x+9}$.

Hint: What limits should you compute? Are you able to compute $\lim_{x \to 3^-} k(x)$? What about $\lim_{x \to 3^+} k(x)$? Does $\lim_{x \to 3} k(x)$ exist?

What is $\lim_{x \to -\infty} k(x)$? $\lim_{x \to \infty} k(x)$? Divide top and bottom by x, and then compute the last 2 limits! As we did in class with other functions!

Exercise 5. Did you memorize Chain rule? Are you able to apply chain rule?

Exercise 6. Find the derivative of the function $f(x) = \tan(2010x) - \cos(89x) - x\sin(x)$. Hint: recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Hint: note that (by Quotient rule and by trig identity) $\{\tan(2010x)\}' = \{\frac{\sin(2010x)}{\cos(2010x)}\}' = \frac{2010\cos(2010x)\cos(2010x) - \sin(2010x)\{-\sin(2010x)2010\}}{\cos^2(2010x)} = \frac{2010}{\cos^2(2010x)}$; note that $\{-\cos(89x)\}' = -\{-\sin(89x)89\}$; note that by product rule one has that $\{-x\sin(x)\}' = -\sin(x) - x\cos(x)$. Can you now glue together all pieces (using sum rule)?

Exercise 7. Find the derivative of the function $h(x) = \tan(x) + \cos(2010x) + \sin(2x)$. Hint: recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Exercise 8. Find the derivative of $h(x) = \frac{\cos(7x)}{e^{7x}}$.

Hint: Use quotient rule and compute yourself: $\frac{-\sin(7x)7e^{7x} - \cos(7x)e^{7x}7}{e^{14x}}$. Of course you must know that 2 times 7 = 14.

Exercise 9. Solve the equation $e^{89x+1} = 2$.

Solution: Apply ln to both sides and get $89x + 1 = \ln(2)$. Can you isolate x?

Exercise 10. Solve the equation $\ln(9x) = 3$.

Solution: Use the laws: $\ln(9) + \ln(x) = 3$, thus $\ln(x) = 3 - \ln(9)$, thus $x = e^{3-\ln(9)}$. Now use your famous calculator!

Exercise 11. Find (if any) the inflection point(s) of $f(x) = e^{2x}(-x^2 + 1)$.

Hint: Are you able to compute the first derivative? Did you get by product rule and chain rule and power rule that $f'(x) = e^{2x}\{2-2x^2-2x\}$? OK! Are you able to get by the same rules the second derivative (that's what you need!)? Did you get that $f''(x) = e^{2x}\{-4x^2-8x+2\}$? Now just use the quadratic formula(do you really know it?) to get the two inflection points! Are you able to find when is f concave down? Concave up?