MAT 1339 A **Assignment 2** (Due TUE. NOV. 2nd, 11:30) **Student Number**:

Name:

Problem 1: Find the equation of the tangent line to the graph of $f(x) = x^2 \sin(2x)$ at $x = \pi$.

Work: Compute $f'(x) = 2x\sin(2x) + x^2\cos(2x)2$ by product rule (and chain rule). Plug in and get $f'(\pi) = 0 + 2\pi^2 = 2\pi^2$. Let $y = mx + n$ be the equation of the tangent line. Then $m = f'(\pi) = 2\pi^2$. Thus $y = 2\pi^2 x + n$.

Note that $f(\pi) = 0$, hence $0 = 2\pi^2\pi + n$. Hence $n = -2\pi^3$. We got $y = 2\pi^2 x - 2\pi^3$

Problem 2: Using the rules of differentiation find the derivative of $g(x) = \frac{\cos(2x)+x-2}{\sin(2x)}$.

Work: By quotient rule and chain rule one has that $g'(x) = \frac{\{-\sin(2x)2+1\}\sin(2x)-\{\cos(2x)+x-2\}\cos(2x)2}{\sin^2(2x)}$.

No need for simplifications!

Problem 3: If $g(x) = 5\sin(4\cos(3x))$ find the derivative of $g(x)$.

Work: Using 2 times chain rule one gets: $g'(x) = 5\cos(4\cos(3x))4(-\sin(3x))3 = -60\cos(4\cos(3x))\sin(3x)$.

Problem 4: If $f(x) = ax^3 + bx^2 + cx + d$ find the values of a, b, c, d such that:

$(0, 1)$ is a point of inflection for f;

$(2, 6)$ is a local maximum.

Work: From $f(0) = 1$ one gets that $d = f(0) = 1$. Hence $f(x) = ax^3 + bx^2 + cx + 1$. Next compute $f'(x) = 3ax^2 + 2bx + c$. From $f'(2) = 0$, one gets that $12a + 4b + c = 0$. Compute now $f''(x) = 6ax + 2b$. From $f''(0) = 0$ one gets $b = 0$.

Hence $12a + c = 0$. FROM $f(2) = 6$ one gets $8a + 2c + 1 = 6$, i.e., $8a + 2c = 5$.

Hence $24a + 2c = 0$ AND $8a + 2c = 5$. SUB and get

$16a = -5$, hence $a = \frac{-5}{16}$. Thus we got $c = -12a = \frac{15}{4}$.

Problem 5: If $f(x) = 6x^2 - 11x - x^3$ find the critical numbers. Then find the absolute maximum and minimum values on the interval $0 \le x \le 4$.

Work: Compute $f'(x) = 12x - 11 - 3x^2$. For critical numbers solve: $f'(x) = 0$, i.e., $-3x^2 + 12x - 11 = 0$. Using the quadratic formula one has:

$x_1 = \frac{-12-\sqrt{144-4(-3)(-11)}}{-6} = \frac{6+\sqrt{3}}{3}$, and hence by symmetry $x_2 = \frac{6-\sqrt{3}}{3}$.

Now note that $f(0) = 0$ and $f(4) = -12$.

Plug in and get $f(\frac{6+\sqrt{3}}{3}) \simeq -5.615$ and $f(\frac{6-\sqrt{3}}{3}) \simeq -6.385$.

Hence the absolute maximum value is 0, and the absolute minimum value is -12.

Problem 6: Let $g(x) = \frac{7x-3}{9-6x+x^2}$. Find all vertical asymptotes, horizontal asymptotes (if any).

Work: VA: Note that $9-6x+x^2 = (3-x)^2$, hence we compute $\lim_{x\to 3^-} \frac{7x-3}{9-6x+x^2} = +\infty$, and $\lim_{x\to 3^+} \frac{7x-3}{9-6x+x^2} = +\infty$.

Conclusion: $x = 3$ is a vertical asymptote!

HA: We compute $\lim_{x\to\infty} \frac{7x-3}{9-6x+x^2} = \lim_{x\to\infty} \frac{7-\frac{3}{x}}{\frac{9}{x}-6+x} = 0$ and $\lim_{x\to-\infty} \frac{7x-3}{9-6x+x^2} = \lim_{x\to-\infty} \frac{7-\frac{3}{x}}{\frac{9}{x}-6+x} = 0$.

Conclusion: $y = 0$ is a horizontal asymptote!