MAT 1339 A **Assignment 3** (Due THU. NOV. 11th, 11:30) **Student Number**:

Name:

Problem 1: Find the equation of the tangent line to the graph of $f(x) = \cos(e^{x-5} - 1) + x$ at the point $(5, 6)$. Hint: Recall that such an equation has the form $y = mx + n$. What is the meaning of m? Find m and n.

Work: Say that the equations is $y = mx + n$. Then $f'(x) = \{-\sin(e^{x-5} - 1)\}(e^{x-5}) + 1$. Thus $m = f'(5) = 1$. So $y = x + n$.

Since $6 = 5 + n$ we get that $n = 6 - 5 = 1$. Thus $y = x + 1$ is the equation we are looking for.

Problem 2: Using the rules of differentiation find the derivative of (i) $89^{7x-2010}$;

(ii) $37^{37x-x^2}e^{2x}$;

(iii) $\frac{88^{2x}}{38^{4x}}$.

Hint: do not simplify!

Work: (i) By chain rule one has that the derivative is $\{89^{7x-2010}\ln(89)\} \times 7$.

(ii) By product rule and chain rule one sees that the derivative is $\{37^{37x-x^2}\ln(37)\}(37-2x)e^{2x} + 37^{37x-x^2}2e^{2x}$.

(iii) By quotient rule one sees that the derivative is $\frac{88^{2x}\cdot\ln(88)\cdot 2\cdot 38^{4x} - 88^{2x}\cdot 38^{4x}\cdot\ln(38)\cdot 4}{38^{8x}}$.

Problem 3: Superwoman is pulling a batmobile from a ditch. The tension in the cable is $20000N$ at an angle of 40 degrees to the horizontal. Find the magnitudes of the vertical and horizontal components of the force.

Work: (see page 348 for a nice picture!) The computations are as follows:

Say that F_v and F_h are the 2 rectangular components. In the triangle involving F_v and F_h and the tension we notes that:

$|F_h| = 20000\cos(40) = 15320.8888$

and

$|F_v| = 20000\sin(40) = 12855.7521$

We used degrees!

Problem 4: What is the magnitude of $\frac{(3\overrightarrow{v}+\overrightarrow{u})-\overrightarrow{v}-\overrightarrow{v}-\overrightarrow{v}}{4|\overrightarrow{u}|}$ for any non-zero vectors $\overrightarrow{u}$ and $\overrightarrow{v}$?

Work: Note that our vector is in fact $\frac{3\overrightarrow{v}+\overrightarrow{u}-\overrightarrow{v}-\overrightarrow{v}-\overrightarrow{v}}{4|\overrightarrow{u}|} = \frac{\overrightarrow{u}}{4|\overrightarrow{u}|}$ (by using commutativity!)

So its size is $\frac{\overrightarrow{u}}{4|\overrightarrow{u}|} = 1/4 = 0.25$.

Problem 5: In a hexagon $ABCDEF$, opposites sides are equal and parallel, and moreover $2\overrightarrow{AB} = \overrightarrow{FC}$. Express $\overrightarrow{BF}$ in terms of $\overrightarrow{FA}$ and $\overrightarrow{BA}$. Hint: draw a picture!

Work: Note that $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF} = \overrightarrow{BA} - \overrightarrow{FA}$.