Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 6

Due date: Oct. 28

Instructor (circle one): Jing Li , Catalin Rada , Frithjof Lutscher

DGD (circle one): 1 , 2 , 3 , 4

Student Name (printed): _____

Student ID Number: _____

Question 1

An animal species is described by the discrete dynamical system

$$N_{i+1} = 1.5N_i(1 - N_i) - hN_i \quad , \quad i = 0, 1, 2, 3, \ldots,$$

where N_i is the fraction of the maximal population after i years. h is the harvesting effort of the predators.

a) Find the equilibrium points. One of these points will depend of h.

Equilibrium points : [ ]

b) Given the largest interval for h such that the equilibrium points in (**a**) have a biological meaning.

Interval for h : [ ]

c) Find the equilibrium harvest R for this species as a function of h.

$R =$ [ ]

d) Determine the harvesting effort h that will maximize the equilibrium harvest.

$h =$ [ ]

e) Give the maximal equilibrium harvest.

$R =$ [ ]

f) Is the maximal equilibrium harvest stable or unstable ?

The maximal equilibrium harvest is (circle one) stable or unstable because

[ ].

Question 2

The Law of mass action states that the rate of a chemical reaction between reactants is proportional to the product of the concentrations of the reactants.

Consider the chemical reaction

$$A + 2B \to AB_2$$

between two chemical reactants A and B. If a is the initial concentration of the reactant A and b is the initial concentration of the reactant B, then we get from the law of mass action that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(a-x)(b-2x)^2 \ ,$$

where x is the concentration of the product AB_2 of the chemical reaction and k is a constant of proportionality. The concentration is the number of molecules per unit of volume. We have that $0 \leq x \leq \min\{a, b/2\}$ (the minimum of a and $b/2$) because the chemical reaction stops when one of the reactant is exhausted. We assume in the model above that no reactant is added to the system after the beginning of the chemical reaction.

If $a = 2$ and $b = 3$, find the concentration x of the product of the reaction when the rate of the reaction is the fastest. Note that the answer is independent of the value of k.

$x =$ [ ]

Question 3

Use a cubic Taylor polynomial to approximate the value of $\ln(1.01)$. The Taylor polynomial is

$p(x) =$ [ ]

and $\ln(1.01) \approx$ [ ].

Question 4

For each of the limits below, determine if it is possible to use the Hospital's rule to evaluate the limit and, if it is possible, evaluate the limit.

a) $\displaystyle\lim_{x\to 0} \frac{4-\sqrt{3x^2+16}}{3x}$ **b**) $\displaystyle\lim_{x\to\infty} \frac{e^x+1}{x^2}$

c) $\displaystyle\lim_{x\to 0} x^{-1/3}\sin(x)$ **d**) $\displaystyle\lim_{x\to 1} \frac{2-x}{\ln(2-x)}$

a) We (circle one) may or may not use the Hospital's rule and $\displaystyle\lim_{x\to 0} \frac{4-\sqrt{3x^2+16}}{3x} =$ [ ].

b) We (circle one) may or may not use the Hospital's rule and $\displaystyle\lim_{t\to\infty} \frac{e^x+1}{x^2} =$ [ ].

c) We (circle one) may or may not use the Hospital's rule and $\displaystyle\lim_{t\to 0} x^{-1/3}\sin(x) =$ [ ].

d) We (circle one) may or may not use the Hospital's rule and $\displaystyle\lim_{x\to 1} \frac{2-x}{\ln(2-x)} =$ [ ].