Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 4

Question 1
Compute the derivative of the following functions.

a) $f(x) = -\cos^2(1-3x)$ **b**) $h(t) = \dfrac{1}{\sin(3t^2)}$

c) $g(x) = \dfrac{x - e^{-2x}}{1 - xe^{-2x}}$ **d**) $f(y) = \cos(\sqrt{y^2+4})$

Don't forget to simplify the results as much as possible.

Solution:
a) $f(x) = h_1(h_2(h_3(x)))$, where $h_3(x) = 1-3x$, $h_2(x) = \cos(x)$ and $h_1(x) = -x^2$. We get

$$f'(x) = -2cos(1-3x)\frac{\mathrm{d}}{\mathrm{d}x}\Big(\cos(1-3x)\Big) = 2cos(1-3x)\sin(1-3x)\frac{\mathrm{d}}{\mathrm{d}x}(1-3x)$$
$$= -6\cos(1-3x)\sin(1-3x)\ .$$

b) We may use the rule for the derivative of a quotient to get

$$h'(t) = \frac{\left(\frac{\mathrm{d}}{\mathrm{d}t}(1)\right)\sin(3t^2) - \frac{\mathrm{d}}{\mathrm{d}t}\sin(3t^2)}{\sin^2(3t^2)} = \frac{-\cos(3t^2)\frac{\mathrm{d}}{\mathrm{d}t}(3t^2)}{\sin^2(3t^2)}$$
$$= \frac{-6t\cos(3t^2)}{\sin^2(3t^2)}\ .$$

We also have that $h(t) = h_1(h_2(h_3(t)))$, where $h_3 = 3t^2$, $h_2(x) = \sin(x)$ and $h_1(x) = x^{-1}$. Thus,

$$h'(t) = -\sin^{-2}(3t^2)\frac{\mathrm{d}}{\mathrm{d}t}\sin(3t^2) = -\sin^{-2}(3t^2)\cos(3t^2)\frac{\mathrm{d}}{\mathrm{d}t}(3t^2) = \ = -6t\cos(3t^2)\sin^{-2}(3t^2)\ .$$

c) We may use the rule for the derivative of a quotient to get

$$g;(x) = \frac{\left(\frac{\mathrm{d}}{\mathrm{d}x}(x-e^{-2x})\right)(1-xe^{-2x}) - (x-e^{-2x})\frac{\mathrm{d}}{\mathrm{d}x}(1-xe^{-2x})}{(1-xe^{-2x})^2}$$
$$= \frac{(1+2e^{-2x})(1-xe^{-2x}) - (x-e^{-2x})(-e^{-2x}+2xe^{-2x})}{(1-xe^{-2x})^2}$$
$$= \frac{(1+2e^{-2x}-xe^{-2x}-2xe^{-4x}) - (-xe^{-2x}+e^{-4x}+2x^2e^{-2x}-2xe^{-4x})}{(1-xe^{-2x})^2}$$
$$= \frac{1+2e^{-2x}-e^{-4x}-2x^2e^{-2x}}{(1-xe^{-2x})^2}\ .$$

d) We have $f(y) = h_1(h_2(h_3(y)))$, where $h_1(y) = \cos(y)$, $h_2(y) = \sqrt{y} = y^{1/2}$ and $h_3(y) = y^2+4$. Hence,

$$\begin{aligned} f'(y) &= -\sin\left((y^2+4)^{1/2}\right)\frac{\mathrm{d}}{\mathrm{d}y}(y^2+4)^{1/2} = -\frac{1}{2}\sin\left((y^2+4)^{1/2}\right)(y^2+4)^{-1/2}\frac{\mathrm{d}}{\mathrm{d}y}(y^2+4) \\ &= -\frac{1}{2}\sin\left((y^2+4)^{1/2}\right)(y^2+4)^{-1/2}(2y) \\ &= -y(y^2+4)^{-1/2}\sin\left((y^2+4)^{1/2}\right) . \end{aligned}$$

Question 2
If $f(x) = x + 2e^x$, find the value of $g'(1+2e)$, where $g(x) = f^{-1}(x)$ for all x.

Solution:
If we differentiate both sides of the equality $g(f(x)) = x$, we get

$$g'(f(x))f'(x) = 1 \Rightarrow g'(f(x)) = \frac{1}{f'(x)} .$$

Since $f'(x) = 1 + 2e^x$, we get

$$g'(f(1)) = \frac{1}{f'(1)} \Rightarrow g'(1+2e) = \frac{1}{1+2e} .$$

Question 3
Find the equation of the tangent line to the curve $y = xe^x\cos(x)$ at $x = \pi$.

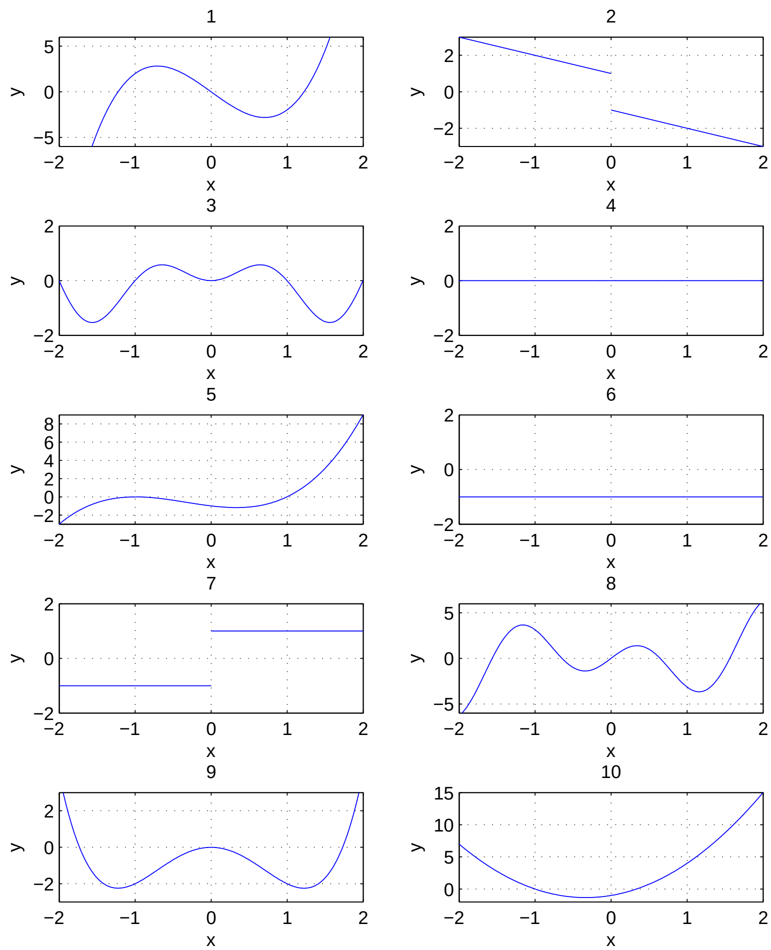
Solution:
Let $y = f(x) = xe^x\cos(x)$. Thus,

$$f'(x) = e^x\cos(x) + xe^x\cos(x) - xe^x\sin(x) = (\cos(x) + x\cos(x) - x\sin(x))e^x .$$

Since $f'(\pi) = (-1-\pi)e^\pi$ and $f(\pi) = -\pi e^\pi$, the equation of the tangent line to the curve at $(\pi, f(\pi)) = (\pi, -\pi e^\pi)$ is

$$y = f(\pi) + f'(\pi)(x-\pi) = -\pi e^\pi + (-1-\pi)e^\pi(x-\pi) .$$

Question 4
A clumsy mathematician drops on the floor the following figures.



Five of the figures represent the graphs of functions and the other five represent the graphs of the derivative of these functions. Write down the pairs (n_1, n_2), where n_1 is the number of the figure associated to the graph of a fonction f and n_2 is the number of the figure associated to the graphe of the derivative f' of the function. **You cannot use a figure more than once**.

Solution:

n_1	n_2
5	10
9	1
3	8
2	6
7	4

The pair (6,4) is acceptable but with this choice, you will not be able to use all 10 figures.