Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 3

Due date: Oct. 7

Instructor (circle one): Jing Li , Catalin Rada , Frithjof Lutscher

DGD (circle one): 1 , 2 , 3 , 4

Student Name (printed): _____

Student ID Number: _____

Question 1

To estimate the limit $\lim_{x\to 0} f(x)$, where $f(x) = \dfrac{e^x - 1 - x}{x^2}$, one may use sequences of numerical values for x approaching 0.

a) Give two sequences to estimate the limit. A few terms for each sequence is enough.

$x_n = 1/n$	$f(x_n)$
1	0.71828
1/2	0.594885
1/3	0.56051
1/4	0.5444
⋮	⋮
1/1000	0.5001667
⋮	⋮
1/10000	0.5000167
↓	↓
0	0.5

and

$x_n = -1/n$	$f(x_n)$
−1	0.367879
−1/2	0.426122
−1/3	0.448781
−1/4	0.4608125
⋮	⋮
−1/100	0.499833
⋮	⋮
−1/1000	0.499983
↓	↓
0	0.5

b) We may conclude that $\lim_{x\to 0} f(x) =$ $\boxed{0.5}$.

Question 2

Does the limit $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$ exist ? Answer : $\boxed{\text{no}}$

Justify your answer in one line.

$$\boxed{\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{2-x}{x-2} = -1 \neq \lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1}\ .$$

Question 3

What is the value of the limit $\lim_{x\to 1} \dfrac{x-1}{x^2-6x+5}$? Answer : $\boxed{-1/4}$

Justify your answer without using sequences of numerical values for x.

$$\boxed{\lim_{x\to 1} \frac{x-1}{x^2-6x+5} = \lim_{x\to 1} \frac{x-1}{(x-1)(x-5)} = \lim_{x\to 1} \frac{1}{x-5} = \frac{1}{1-5} = -\frac{1}{4}}\ .$$

Question 4

We consider the function

$$f(x) = \begin{cases} 0 & \text{if} \quad x \in \mathbb{Q} \\ x^2 & \text{if} \quad x \notin \mathbb{Q} \end{cases}$$

a) Is the function f continuous at $x = 0$? Answer : Yes
Justify your answer in one line.

For $x \in \mathbb{Q}$, $f(x) = 0 \to 0 = f(0)$ as $x \to 0$. For $x \notin \mathbb{Q}$, $f(x) = x^2 \to 0 = f(0)$ as $x \to 0$

b) Does the derivative of f exist at $x = 0$? Answer : Yes and $f'(0) = 0$
Justify your answer in one line.

For $x \in \mathbb{Q}$, $f(x) = 0$ and $\dfrac{f(x) - f(0)}{x - 0} = 0 \to 0$ as $x \to 0$. For $x \notin \mathbb{Q}$, $f(x) = x^2$ and $\dfrac{f(x) - f(0)}{x - 0} = \dfrac{x^2 - 0}{x - 0} = x \to 0$ as $x \to 0$.

Note : Try to visualize the graph of f in your mind. Is it a nice continuous curve?