Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 2

Due date: Sept. 30

Instructor (circle one): **Jing Li** , **Catalin Rada** , **Frithjof Lutscher**

DGD (circle one): **1** , **2** , **3** , **4**

Student Name (printed): ______________________________

Student ID Number: ______________________________

Question 1

A model for the change in population size of the zooplankton species **Daphnia galeata mendota** in Base Line Lake in Michigan is given by $N(t) = N_0 e^{rt}$, where $N(t)$ is the population size at time t, N_0 is the initial size of the population and r is the **intrinsic rate of growth**.

a) If the initial size of the population is 200 and the size of the population after one week is 250, find the intrinsic rate of growth of this population.

$r = \boxed{\ln(5/4) = 0.22314\ldots}$

We have $N(1) = 250$, $N_0 = 200$ and $t = 1$. Thus, $N(t) = N_0 e^{rt}$ yields

$$250 = 200 e^r \Rightarrow \frac{5}{4} = e^r \Rightarrow r = \ln\left(\frac{5}{4}\right) .$$

b) If the size of the population is 2.5 times its initial size after 2 weeks, find the intrinsic rate of growth of this population.

$r = \boxed{\frac{\ln(2.5)}{2} = 0.458145\ldots}$

We have $N(2) = 2.5N_0$ and $t = 2$. Thus, $N(t) = N_0 e^{rt}$ yields

$$2.5N_0 = N_0 e^{2r} \Rightarrow 2.5 = e^{2r} \Rightarrow \ln(2.5) = 2r \Rightarrow r = \frac{\ln(2.5)}{2} .$$

c) If the intrinsic rate of growth is 0.95, what was the initial size of the population if the size of the population is 200 after 3 weeks?

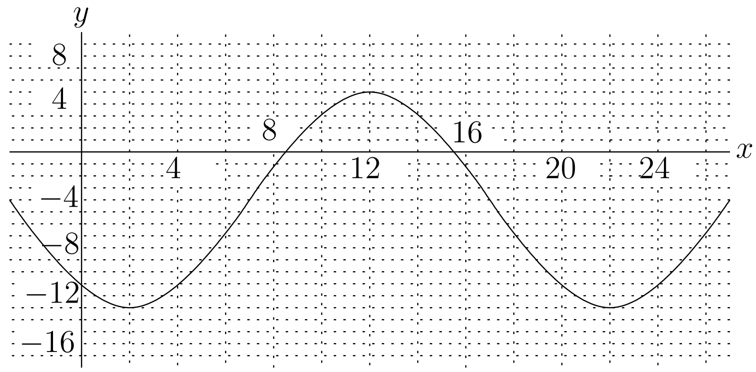
$N_0 = \boxed{= \frac{200}{e^{2.85}} = 11.56886\ldots}$

We have $N(3) = 200$, $r = 0.95$ and $t = 3$. Thus, $N(t) = N_0 e^{rt}$ yields

$$200 = N_0 e^{3\times 0.95} \Rightarrow N_0 = \frac{200}{e^{3\times 0.95}} \Rightarrow N_0 = \frac{200}{e^{2.85}} .$$

Question 2

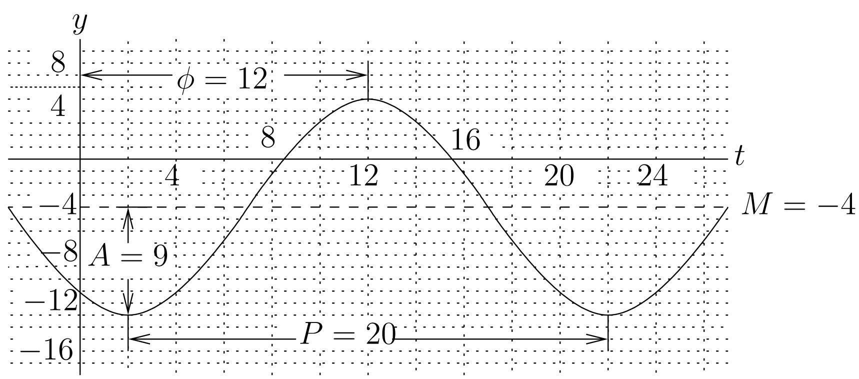
A sinusoidal function f has the following graph.



$y = f(x)$

Give a formula to define f.

$f(x) = \boxed{-4 + 9\cos\left(\frac{\pi}{10}(t-12)\right)}$



$y = f(t)$

The period is $P = 20$, the phase is $\phi = 12$, the mean is $M = -4$ and the amplitude is $A = 9$. Thus,

$$f(t) = M + A\cos\left(\frac{2\pi}{P}(t-\phi)\right) = -4 + 9\cos\left(\frac{2\pi}{20}(t-12)\right) = -4 + 9\cos\left(\frac{\pi}{10}(t-12)\right) .$$

Question 3

The activity level of mosquitoes over a swamp is measured by the number of mosquitoes by cubic meter. It is governed by a sinusoidal function. The maximum level of activity is a 19 :00 when there are about 10 mosquitoes per cubic meter and the lowest level is at 7 :00 when there are no mosquitoes per cubic meter.

a) Find the function f that governs the activity level of the population of mosquitoes as a function of the time t during the day.

$f(t) = \boxed{5 + 5\cos\left(\frac{\pi}{12}(t-19)\right)}$

The period is $P = 24$ hours, the mean is $M = 5$ mosquitos per cubic meter, the amplitude is $A = 5$ mosquitos per cubic meter and the phase is $\phi = 19$ hours. Thus,

$$f(t) = M + A\cos\left(\frac{2\pi}{P}(t-\phi)\right) = 5 + 5\cos\left(\frac{2\pi}{24}(t-19)\right) = 5 + 5\cos\left(\frac{\pi}{12}(t-19)\right) .$$

b) Draw the graph of the activity level of the population of mosquitoes as a function of the time.