Comments on lecture 1

1.5 DISCRETE-TIME DYNAMICAL SYSTEMS

The source for this comments is your textbook!!!!

Part I.

DEF: • A discrete-time dynamical system describes the relation between a quantity measured at the beginning and at the end of an experiment OR a time interval;

• If the measurement is represented (or denoted) by (the variable m), then m_t denotes the measurement at the beginning of the experiment and m_{t+1} denotes the measurement at the end of the experiment;

• The relation between m_t and m_{t+1} is given by the DISCRETE-TIME DYNAMICAL SYSTEM: $m_{t+1} = f(m_t)$, where f is called the UPDATING FUNCTION.

Example 0.0.1. ***Bacterial population:*** *Read example 1.5.1/page 53*

Example 0.0.2. ***Tree Growth:*** *Read example 1.5.2/page 54*

Example 0.0.3. ***Medication concentration:*** *Read example 1.5.4/page 55*

Part II.

Dealing with Updating Functions (from the point of view of Algebra)

a) Consider the DTDS: $m_{t+1} = f(m_t)$.

Question: What does the COMPOSITION $f \circ f$ mean?

This: $(f \circ f)(m_t) = f(f(m_t)) = f(m_{t+1}) = m_{t+2}$, SO: the COMPOSITION of an Updating Function with itself corresponds to a 2-step updating function.

Example 0.0.4. ***Bacterial population:*** *Consider the bacterial population; we have* $(f \circ f)(b_t) = f(f(b_t)) = f(2b_t) = 2 \times (2b_t) = 4b_t$, *i.e., after 2 hours the beautiful population is 4 times BIGGER!*

b) What other parts of Algebra may we use? INVERSEs! Consider the DTDS: $m_{t+1} = f(m_t)$.

Question: What does the inverse f^{-1} mean?

THIS: applying f^{-1} to our relation one gets: $f^{-1}(m_{t+1}) = m_t$, that can be viewed as: $m_t = f^{-1}(m_{t+1})$, SO, the INVERSE of an Updating Function corresponds to an

$$UPDATING \quad Function$$

that goes backwards in time!

Example 0.0.5. ***Bacterial population:*** *Consider the bacterial population; we do have:* $b_{t+1} = 2b_t = f(b_t)$. *Solve for the inverse:*

$f(b_t) = y$ *implies that* $2b_t = y$, *which in turn implies that* $b_t = \frac{y}{2}$, *hence* $f^{-1}(b_t) = \frac{b_t}{2}$. *We have a new DTDS* $b_t = f^{-1}(b_{t+1})$, *that can be written as:* $b_t = \frac{b_{t+1}}{2}$.

Part III. **Solutions**

• Recall that a DTDS describes **some** quantity at the end of an experiment/process/measurement as a function of the **same** quantity at the beginning!

• **Question:** What if we were to continue the process/experiment? Think about the bacterial population! The population will double again, again, and again.....

• To describe a process that is repeated many times we let:

— m_0 = measurement at the beginning;

— m_1 = measurement after one time step;

— m_2 = measurement after 2 time steps;

— . . .

— m_t = measurement after t seconds/years/hours/days (or whatever unit one may use) after the beginning of the process/experiment

• **DEFINITION** The SOLUTION of the DTDS: $m_{t+1} = f(m_t)$ is the sequence of values of m_t for $t = 0, 1, 2, 3, \ldots$, STARTING from the INITIAL CONDITION m_0.

NB: We know where we started the process!

• The **GRAPH** of a Solution is a discrete set of points: the time t on the x-axis; m_t on the y-axis:

$(0, m_0), (1, m_1), (2, m_2), \ldots$

• **Example** Consider the bacterial population: $b_{t+1} = 2b_t$ WITH $b_0 = 1.0$ (in millions). We do have: $b_1 = 2b_0 = 2 \times 1 = 2$; $b_2 = 2b_1 = 2 \times 2 = 4$, $b_3 = 2b_2 = 2 \times 4 = 8$, etc...

BUT: $b_1 = 2b_0$; $b_2 = 2b_1 = 2 \times 2 \times b_0 = 2^2 b_0$, $b_3 = 2b_2 = 2 \times 2^2 \times b_0 = 2^3 b_0$, so $b_{t+1} = 2^{t+1} b_0$ — the population after $t + 1$ hours; of course the population after t hours is $b_t = 2^t b_0$

Graph it!

Do: 14, 24, 20 on 64-65!

THE LOCATION and DATE OF DIAGNOSTIC TEST Were CHANGED!

GO TO THE WEB PAGE AND READ THE PIECES OF INFORMATION GIVEN THERE!

Did you do 14, 24, 20 on 64-65? Please attend the dgds!

Comments on lecture 2

Comments on lecture 3