University of Ottawa
MAT 1330B Euler Method Problem
December 1, 2009 Instructor: Catalin Rada

Question 1. Apply Euler Method to the following differential equation to estimate the solution at $t = 1$ starting from the given initial condition. Use $\Delta t = 0.25$:

$\frac{dx}{dt} = e^t + t + 2009$, $x(0) = 1$.

Solution: Note that $x(0) = 1$. Next we get $x(0 + 0.25) = x(0.25) \cong \widehat{x}(0.25) = x(0) + x'(0)\Delta t = 1 + (e^0 + 0 + 2009)(0.25) = 1 + 2010(0.25) = 503.5$.

Now: $x(0.25 + 0.25) = x(0.50) \cong \widehat{x}(0.50) = x(0.25) + x'(0.25)(0.25) \cong 503.5 + (e^{0.25} + 0.25 + 2009)(0.25) = 1006.133506$.

So: $x(0.50+0.25) = x(0.75) \cong \widehat{x}(0.75) = x(0.50)+x'(0.50)(0.25) \cong 1006.133506+(e^{0.50} + 0.50 + 2009)(0.25) = 1508.920686$

Thus $x(0.75 + 0.25) = x(1) \cong \widehat{x}(1) = x(0.75) + x'(0.75)(0.25) \cong 1508.920686 + (e^{0.75} + 0.75 + 2009)(0.25) = 2011.887436$.

Compare with the true value that is obtained by plugging 1 in $x(t) = e^t + \frac{t^2}{2} + 2009t + C$. Since $x(0) = 1$ one gets $C = 0$, so $x(1) = e + \frac{1}{2} + 2009 = 2012.218282$.