Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 1

Due date: Sept. 23

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DGD (circle one): 1 , 2 , 3 , 4

Student Name (printed): _____

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Question 1

The size of a bird population on an island located close to the cost of Newfoundland depends on the local growth rate (birth minus death) and the migration between the island and Newfoundland. A discrete dynamical system modelling this population is

$$x_{n+1} = 0.85x_n + 75 \quad , \quad n = 0, 1, 2, 3, \ldots ,$$

where x_n is the size of the bird population on the island after n years. 0.85 is the local growth rate and 75 is the yearly increase due to migration.

a) If $x_0 = 200$, then

$x_1 =$ [ ] , $x_2 =$ [ ] , $x_3 =$ [ ]

b) Give the updating function f of the dynamical system. $f(x) =$ [ ]

c) Find the equilibrium point x^* of the dynamical system. $x^* =$ [ ]

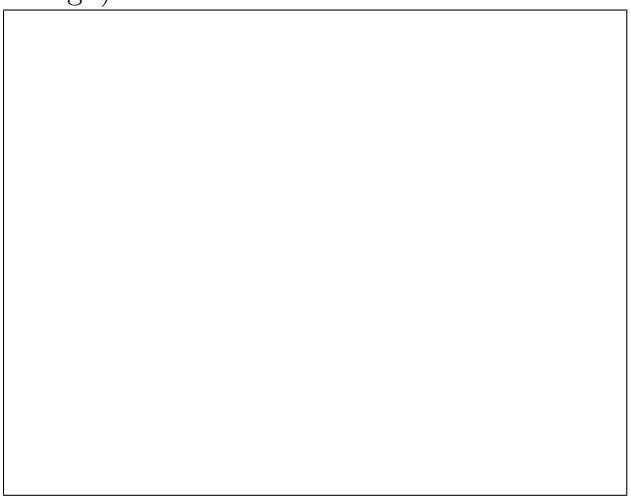
d) Give the solution of the dynamical system with $x_0 = 200$.

$x_n =$ [ ]

e) Draw the solution of the dynamical system with $x_0 = 200$ (four points are enough).

[ ]

f) Draw the cobweb diagram of the dynamical system with $x_0 = 200$ (four iterations are enough).



g) Determine the stability of the equilibrium point using the cobweb diagram.

The equilibrium point is $\boxed{}$

Question 2

The dynamical system

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta x_n} \quad , \quad n = 0, 1, 2, 3, \ldots$$

plays a role in the analysis of nonlinear models of gene and neural networks. α and β are positive parameters. Suppose that $\alpha = 2$ and $\beta = 1$,

a) If $x_0 = 2.5$, then

$x_1 = \boxed{}$, $x_2 = \boxed{}$, $x_3 = \boxed{}$

b) Find the equilibrium points x^* of the dynamical system. $x^* = \boxed{}$