Calculus for the Life Science I
MAT1330A , MAT1330B, MAT1330E
Assignment 1

Due date: Sept. 23

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Student Name (printed): _____

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Question 1

The size of a bird population on an island located close to the cost of Newfoundland depends on the local growth rate (birth minus death) and the migration between the island and Newfoundland. A discrete dynamical system modelling this population is

$$x_{n+1} = 0.85x_n + 75 \quad , \quad n = 0, 1, 2, 3, \ldots ,$$

where x_n is the size of the bird population on the island after n years. 0.85 is the local growth rate and 75 is the yearly increase due to migration.

a) If $x_0 = 200$, then
$x_1 =$ $\boxed{245}$, $x_2 =$ $\boxed{283.25}$, $x_3 =$ $\boxed{315.7625}$

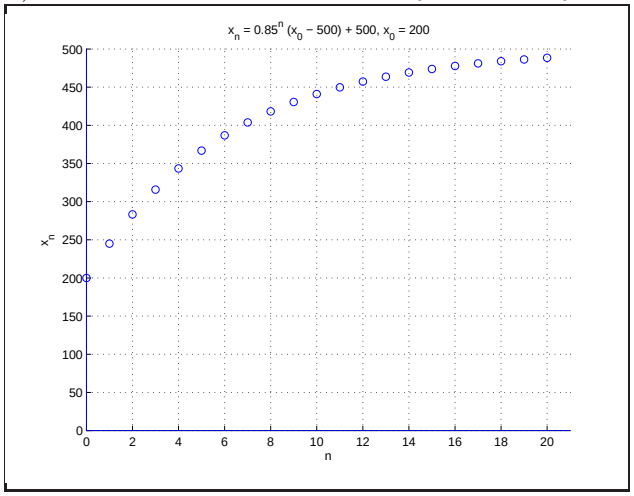
b) Give the updating function f of the dynamical system. $f(x) =$ $\boxed{0.85x + 75}$

c) Find the equilibrium point p of the dynamical system. $p =$ $\boxed{500}$

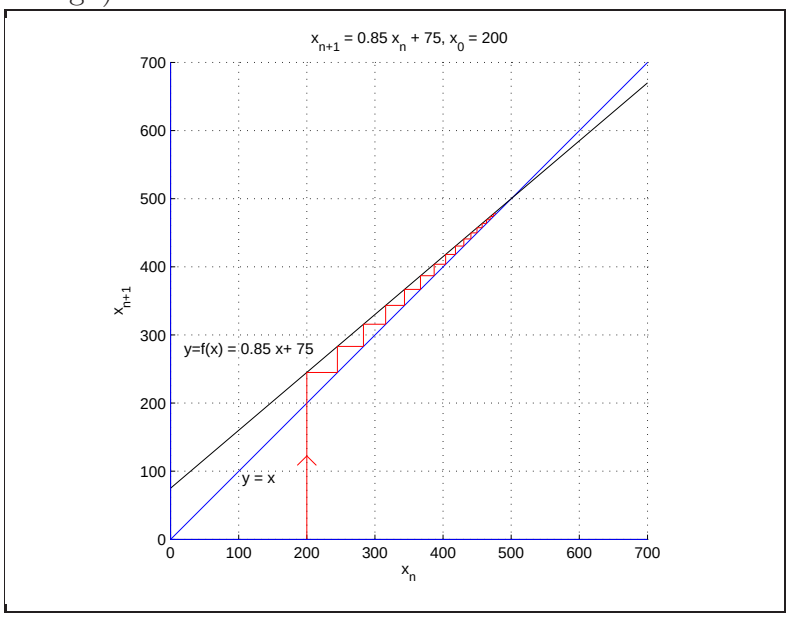
$$p = 0.85p + 75 \Rightarrow p - 0.85p = 75 \Rightarrow 0.15p = 75 \Rightarrow p = 75/0.15 = 500 \ .$$

d) Give the solution of the dynamical system with $x_0 = 200$.
$x_n =$ $\boxed{0.85^n(x_0 - 500) + 500 = -300 \times 0.85^n + 500}$

e) Draw the solution of the dynamical system with $x_0 = 200$ (four points are enough).



f) Draw the cobweb diagram of the dynamical system with $x_0 = 200$ (four iterations are enough).



g) Determine the stability of the equilibrium point using the cobweb diagram.
The equilibrium point is $\boxed{\text{stable}}$ because all orbits converge to this equilibrium point. Moreover, we have a dynamical system of the form $x_{n+1} = rx_n + b$ with $r = 0.85$. So, the system is stable because $|r| < 1$.

Question 2
The dynamical system

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta x_n} \quad , \quad n = 0, 1, 2, 3, \ldots$$

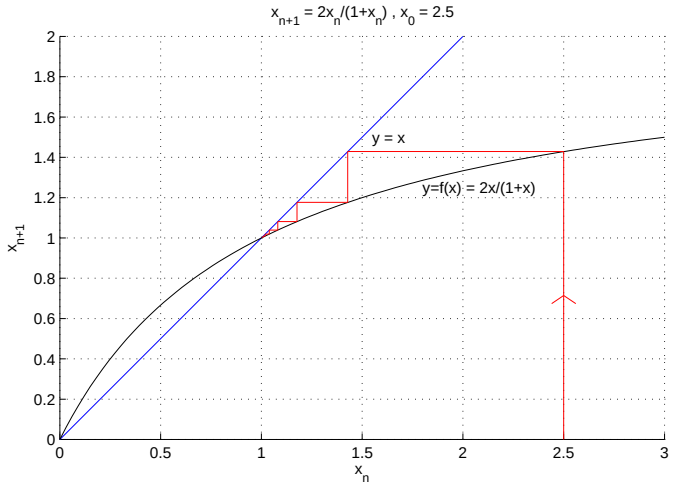
plays a role in the analysis of nonlinear models of gene and neural networks. α and β are positive parameters. Suppose that $\alpha = 2$ and $\beta = 1$,

a) If $x_0 = 2.5$, then
$x_1 = \boxed{1.4286}$, $x_2 = \boxed{1.1765}$, $x_3 = \boxed{1.0811}$

b) Find the equilibrium points p of the dynamical system. $p = \boxed{0 \text{ or } 1}$

$$p = \frac{2p}{1+p} \Rightarrow p(1+p) = 2p \Rightarrow p^2 + p = 2p \Rightarrow p^2 - p = 0 \Rightarrow p(p-1) = 0 \Rightarrow p = 0, 1$$

For you information, the cobweb diagram with $x_0 = 2.5$ is given below.


x_{n+1} = 2x_n/(1+x_n) , x_0 = 2.5
y = x
y=f(x) = 2x/(1+x)
x_{n+1}
x_n
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.5
1.5
2.5
3