MAT 1322C – Assignment 1 (due on Monday January 24, 17:30pm)

Last Name (please print): First Name:

1. Use the Comparison Test to determine if the integral converges or diverges. Justify your answer in a clear manner.

(i) $\int_1^\infty \frac{1}{\sqrt{x}+x^4}\,dx\,,$ (ii) $\int_0^1 \frac{e^x}{x^3}\,dx\,.$

Solution:

2. Let R be the region bounded by the curves $y = 1/x$, $y = -1/x$ and the vertical lines $x = 1$ and $x = 2$. Denote by S the solid obtained by rotating R about the axis $y = 2$.

(i) Sketch the cross-section of the solid with the xy-plane. Also sketch the cross-section S_x of the solid S with the plane through x $(1 \leq x \leq 2)$ and perpendicular to the x-axis.

(ii) What is the area $A(x)$ of the cross-section S_x?

(iii) Using the result from (ii) calculate the volume of S.

Solution:

3. Let R be the region in the xy-plane determined by the conditions $2 \leq x \leq 3$ and $1/x \leq y \leq 2/x$. Let S be the solid that is obtained by rotating R about the vertical line $x = 1$. Follow the steps below to calculate the volume of S by the method of cylindrical shells.

(i) Sketch the cross-section of the solid with the xy-plane. Also sketch a typical cylindrical shell of inner radius x and outer radius $x + \Delta x$ for $2 \leq x \leq 3$ and Δx small. Include dimensions in your sketches.

(ii) Give an approximation for the volume of the cylindrical shell in (i) as a function of Δx and x.

(iii)Using the result from (ii), calculate the volume of S.

Solution:

4. Sketch the arc $x = e^{-t}\cos(t)$, $y = e^{-t}\sin(t)$, $0 \leq t \leq \pi$ and calculate its exact length. Note: After a simplification the integral is not very complicated.

Solution: