Solutions

MAT 1322 A **Assignment 6** (Due Wed. April 6th at 8:30) **Student Number:**

1. Find the tangent plane to the surface $z = x\ln(y) + x^2y + 2$ at the point $(x, y, z) = (2, 1, 6)$.

Work:

$f_x = \ln y + 2xy \Rightarrow f_x(2,1) = \ln(1) + 2(2)(1) = 4$

$f_y = x/y + x^2 \Rightarrow f_y(2,1) = 2/1 + (2)^2 = 6$

tangent plane is $z = 6 + f_x(2,1)(x-2) + f_y(2,1)(y-1)$

$= 6 + 4(x-2) + 6(y-1)$

Answer: $z = 4x + 6y - 8$

2. Find the linear approximation of the function $f(x, y) = \sqrt{xy^2 + 2}$ at the point $(x, y) = (2, 1)$ and use it to estimate $f(2.02, 0.99)$.

Work:

$f(2,1) = \sqrt{2(1)^2+2} = \sqrt{4} = 2$

$f_x = \frac{1}{2}(xy^2+2)^{-1/2}(y^2) = \frac{y^2}{2\sqrt{xy^2+2}} \Rightarrow f_x(2,1) = \frac{(1)^2}{2\sqrt{4}} = 1/4$

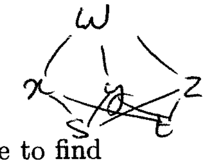
$f_y = \frac{1}{2}(xy^2+2)^{-1/2}(2xy) = \frac{xy}{\sqrt{xy^2+2}} \Rightarrow f_y(2,1) = \frac{2(1)}{\sqrt{4}} = 1$

$L(x,y) = 2 + f_x(2,1)(x-2) + f_y(2,1)(y-1)$

$f(2.02, 0.99) \approx L(2.02, 0.99) = 2 + \frac{1}{4}(0.02) + (-0.01)$

Answers: $L(x, y) = 2 + \frac{1}{4}(x-2) + (y-1)$ $f(2.02, 0.99) \simeq 1.995$

or $L(x,y) = \frac{1}{4}x + y + 1/2$



3. Given that $w = \ln(x^2 + y + z^2)$, $x = \cos(s)$, $y = \dfrac{s}{\pi t}$ and $z = t$, use the Chain Rule to find $\dfrac{\partial w}{\partial t}$ and $\dfrac{\partial w}{\partial s}$ at the point where $t = 1$ and $s = \pi$.

$s = \pi,\ t = 1$

$x = -1$

$y = 1$

$z = 1$

Work:

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= \left(\frac{2x}{x^2+y+z^2}\right)(0) + \left(\frac{1}{x^2+y+z^2}\right)\left(\frac{-s}{\pi t^2}\right) + \left(\frac{2z}{x^2+y+z^2}\right)(1)$$

$$= \left(\frac{1}{3}\right)(-1) + \left(\frac{2}{3}\right)(1)$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

$$= \left(\frac{2x}{x^2+y+z^2}\right)(-\sin s) + \left(\frac{1}{x^2+y+z^2}\right)\left(\frac{1}{\pi t}\right) + \left(\frac{2z}{x^2+y+z^2}\right)(0)$$

$$= \left(\frac{-2}{3}\right)(0) + \left(\frac{1}{3}\right)\left(\frac{1}{\pi}\right) + 0$$

Answers: $\dfrac{\partial w}{\partial t} = \dfrac{1}{3}$ $\qquad$ $\dfrac{\partial w}{\partial s} = \dfrac{1}{3\pi}$

4. Determine $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if z is given implicitly as a function of x and y by the equation $xy^2 + xe^z = z^2$.

Work:

$$\frac{\partial}{\partial x}\left(xy^2 + xe^2\right) = \frac{\partial}{\partial x}(z^2)$$

$$y^2 + e^2 + xe^2\frac{\partial z}{\partial x} = 2z\frac{\partial z}{\partial x}$$

$$\left(2z - xe^2\right)\frac{\partial z}{\partial x} = y^2 + e^2$$

$$\frac{\partial}{\partial y}\left(xy^2 + xe^2\right) = \frac{\partial}{\partial y}(z^2)$$

$$2xy + xe^2\frac{\partial z}{\partial y} = 2z\frac{\partial z}{\partial y}$$

$$\left(2z - xe^2\right)\frac{\partial z}{\partial y} = 2xy$$

Answers: $\dfrac{\partial z}{\partial x} = \dfrac{y^2+e^2}{2z - xe^2}$ $\qquad$ $\dfrac{\partial z}{\partial y} = \dfrac{2xy}{2z - xe^2}$