MAT 1322 C **Assignment 3** (Due Wed. Feb. 16th at 17:30) **Student Number**:

1. Consider the initial value problem $y' = -4xy,\ y(0) = 3$.
(a) Use Euler's Method to estimate $y(1)$ to 4 decimal places, with 10 steps.
(b) What is the true solution of the initial value problem?
(c) What is the true value of $y(1)$ to 4 decimal places?
(d) What is the error of the approximation in (a)?

Solution: **Answers:** (a) $y(1) \approx$ (b) $y(x) =$

(c) $y(1) =$ (d) $\epsilon =$

2. A detective finds a murder victim in a room with constant temperature 21°C. At 5:00am, the body's temperature was 34.2°C. One hour later, it was 31.8°C. Normal body temperature is 37°C. Assume the body's temperature, $B(t)$, follows Newton's Law of Cooling.
(a) Set up and solve the differential equation for $B(t)$.
(b) Estimate the time of the murder.

Solution: **Answers:** (a) $B(t) =$ (b) murder occured at

3. A reservoir contains 500 litres of pure water. Brine (salty water) that contains 0.1 kg/L of salt is added at a rate of 2 L/min. Brine from a second source with 0.05 kg/L of salt is added at a rate of 3 L/min. Assume that the reservoir is instantaneously well-mixed. The reservoir is drained at a rate of 5 L/min. Let $Q(t)$ be the amount of salt (in kg) at time t (in min).
(a) Set up and solve the differential equation for $Q(t)$.
(b) How much salt is there in the reservoir after 4 hours?

Solution: **Answers:** (a) $Q(t) =$ (b) kg

4. A conservation group estimates that a wildlife preserve can sustain a herd of 2000 gnus. They also know that the relative growth rate of gnus would be 0.125 (per year) in an unconstrained environment. A herd of 800 gnus is placed in the preserve.
(a) Give the formula for the number $P(t)$ of gnus after t years, assuming that the population follows the Logistic Model.
(b) How long will it take for the population to grow to 1000 ?

Solution: **Answers:** (a) $P(t) =$ (b) years

5. Do the following sequences converge or diverge? If they converge, give the limit.

(a) $a_n = \dfrac{(-1)^n\, 2n}{n^2 + 3n}$

(b) $a_n = \sin(n + 1)$

(c) $a_n = \dfrac{5n^2 + 6n + 3}{3n^2 + 7}$

(d) $a_n = \dfrac{3^{n+1}}{2^{n+3}}$

Solution: **Answers:** (a) (b)

(c) (d)