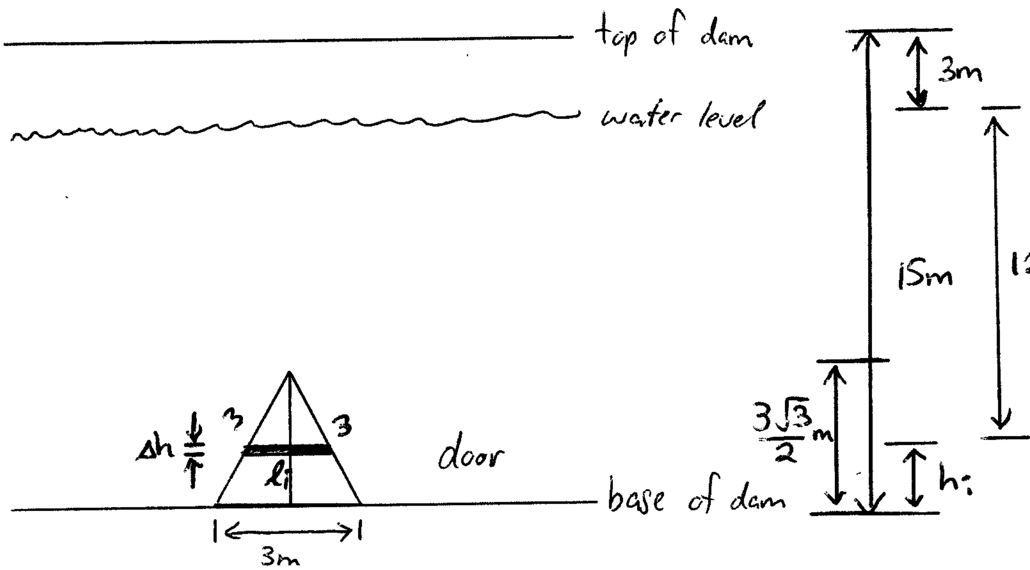
Solutions

MAT 1322 A **Assignment 2** (Due Mon. Jan. 31st at 10:00) **Student Number:**

A dam 15 m high holds the water in a reservoir. For safety reasons, the water level is maintained at a height 3 m below the top of the dam. At the base of the dam, engineers have installed a vertical door in the shape of an equilateral triangle with sides of length 3 m, as shown in the diagram below. Calculate the total hydrostatic force on the door. Your solution must be clear and complete. You must define clearly in words each of the variables used and indicate their meaning on the diagram. The density of water is $\rho = 1000$ kg/m^3 and the acceleration due to gravity is $g = 9.8$ m/s^2.



the height of the door is

$\sqrt{(3)^2-(1.5)^2}$

$= 2.5981$ m

$= \frac{3\sqrt{3}}{2}$ m

Take a horizontal strip of length l_i at height h_i of width Δh. The area of the strip is $l_i \Delta h$.

By similar triangles, $\frac{l_i}{3} = \frac{\frac{3\sqrt{3}}{2}-h_i}{\frac{3\sqrt{3}}{2}} = \frac{3\sqrt{3}-2h_i}{3\sqrt{3}}$, so $l_i = 3 - \frac{2}{\sqrt{3}}h_i$.

The area of the strip is then $A_i = (3-\frac{2}{\sqrt{3}}h_i)\Delta h$.

The pressure on this strip is $P_i = \rho g(12-h_i)$.

And thus, the force on the strip is $F_i = P_i A_i = \rho g(12-h_i)(3-\frac{2}{\sqrt{3}}h_i)\Delta h$.

So the total force on the door is $F \approx \sum_i F_i = \sum_i \rho g (12-h_i)(3-\frac{2}{\sqrt{3}}h_i)\Delta h$.

Take the limit as $h \to 0$ to get:

$$F = \lim_{h\to 0} \sum_i \rho g(12-h_i)(3-\frac{2}{\sqrt{3}}h_i)\Delta h = \int_0^{3\sqrt{3}/2} \rho g (12-h)(3-\frac{2}{\sqrt{3}}h)\,dh$$

$$= (1000)(9.8)\int_0^{3\sqrt{3}/2} \left(36 - (3+\frac{24}{\sqrt{3}})h + \frac{2}{\sqrt{3}}h^2\right)dh$$

$$= 9800\left[36h - \frac{1}{2}(3+\frac{24}{\sqrt{3}})h^2 + \frac{2}{3\sqrt{3}}h^3 \Big|_0^{3\sqrt{3}/2}\right] \approx \boxed{425\,225 \text{ N}}$$